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A self-adaptive nonlinear theta method using discontinuity aware quadrature for solving hyperbolic conservation laws

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Abstract

We present a *discontinuity aware quadrature* (DAQ) rule, and use it to develop implicit *self-adaptive theta* (SATh) schemes for the approximation of scalar hyperbolic conservation laws. Our SATh schemes require the solution of a system of two equations, one controlling the cell averages of the solution at the time levels, and the other controlling the space-time averages of the solution. These quantities are used within the DAQ rule to approximate the time integral of the hyperbolic flux function accurately, even when the solution may be discontinuous somewhere over the time interval. The result is a finite volume scheme using the theta time stepping method, with theta defined implicitly (or self-adaptively). We prove that DAQ is accurate to second order when there is a discontinuity in the solution and third order when it is smooth. We prove that the scheme is unconditionally stable (provided that theta is set to be at least 1/2), satisfies the maximum principle and is total variation diminishing under appropriate monotonicity and boundary conditions. Compared to solutions of finite volume schemes using Crank-Nicolson and backward Euler time stepping, solutions of our scheme often approach the accuracy of the former but without oscillation, and they are numerically less diffuse than the later.