

Numerical methods for partial differential equations on surface

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Abstract

Partial differential equations (PDEs) on surfaces have wide applications in many areas. Solving these PDEs numerically can be complicated and computationally expensive. This work targets applications that use implicit or non-parametric representations of closed surfaces or curves and require numerical solution for minimization problems defined on the surfaces.

We showed that the energy function defined on surfaces can be extended to the energy function defined on the nearby tubular neighborhood that gives the same energy when input the constant-along-normal extension. Furthermore, the extended energy function gives the same minimizer as which the original energy function gives in the sense of restriction on the surface. This new approach connects the original energy function to an extended energy function and provides a good framework to solve PDEs numerically on Cartesian grids. We continue the results in previous work to develop a framework to solve more challenging problems defined on surfaces, such as nonlinear and non-convex energy problems, Hamilton-Jacobi-Bellman equations. Currently, an investigation of applying this methodology to the mean curvature flow problem has been done and some preliminary results will be discussed in this talk.