

## Calculus (Type B. 2 Pages)

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**Part I: Choice.** Select only one answer choice from a list of four choices.

1. (5 pts.)  $\lim_{x \rightarrow 0} x \sin \frac{1}{x} =$

- (A) 0      (B)  $\infty$       (C) does not exist      (D) 1

**Ans: (A)**

2. (5 pts.)  $\lim_{x \rightarrow \infty} x \sin \frac{1}{x} =$

- (A) 0      (B)  $\infty$       (C) does not exist      (D) 1

**Ans: (D)**

3. (5 pts.) If  $f(x)$  is a cubic polynomial function, known  $\lim_{x \rightarrow 1} \frac{f(x)}{x - 1} = 1$  and  $\lim_{x \rightarrow -1} \frac{f(x)}{x + 1} = 1$ ,

$\lim_{x \rightarrow 2} f(x) =$

- (A) 3      (B) 6      (C) 9      (D) does not exist

**Ans: (A)**

4. (5 pts.) If  $f(x) = x|x|$ ,  $f'(0) =$

- (A) does not exist      (B) -1      (C) 1      (D) 0

**Ans: (D)**

5. (5 pts.) If  $f(2) = 4$  and  $f'(2) = -2$ ,  $\lim_{x \rightarrow 2} \frac{2f(x) - xf(2)}{x^2 - 4} =$

- (A) 0      (B) 4      (C) -2      (D) -8

**Ans: (C)**

6. (5 pts.) Continue problem 5.,  $\lim_{\Delta x \rightarrow 0} \frac{f(2 - \Delta x) - f(2 + \Delta x)}{\Delta x} =$

- (A) -4      (B) -2      (C) 2      (D) 4

**Ans: (D)**

7. (5 pts.)  $\frac{d^{107}}{dx^{107}}(\sin x) =$

- (A)  $\cos x$       (B)  $-\cos x$       (C)  $-\sin x$       (D)  $\sin x$

**Ans: (B)**

8. (5 pts.) Let  $g(x) = \begin{cases} -x, & \text{if } x \leq -1 \\ 1 - x^2, & \text{if } -1 < x < 1. \end{cases}$  Which of the following is correct?

(A)  $\lim_{x \rightarrow -1^-} g(x) = -1$       (B)  $\lim_{x \rightarrow -1^+} g(x) = 1$

(C)  $\lim_{x \rightarrow -1^-} g'(x) = -1$       (D)  $\lim_{x \rightarrow -1^+} g'(x) = 1$

**Ans:** (C)

9. (5 pts.) Find  $y'$ ,  $y = (x^2 - 3x + 2)(5x^3 - x^2 + 5)$ .

(A)  $y' = 25x^4 - 64x^3 + 39x^2 + 6x - 15$       (B)  $y' = 5x^4 - 64x^3 + 39x^2 + 6x - 15$

(C)  $y' = 25x^4 - 60x^3 + 39x^2 + 6x - 15$       (D)  $y' = 5x^4 - 60x^3 + 39x^2 + 6x - 15$

**Ans:** (A)

10. (5 pts.) Find  $y'$ ,  $y = \frac{x^2 + 5}{x^2 + 6x}$ .

(A)  $y' = \frac{x^4 + 6x^3 + 5x^2 + 30x}{x^2(x+6)^2}$       (B)  $y' = \frac{6x^2 - 10x - 30}{x^2(x+6)^2}$

(C)  $y' = \frac{4x^3 + 18x^2 + 10x + 30}{x^2(x+6)^2}$       (D)  $y' = \frac{2x^3 - 5x^2 - 30x}{x^2(x+6)^2}$

**Ans:** (B)

## Part II: Fill in the Blanks.

(a) (5 pts.)  $\lim_{x \rightarrow 0} \frac{|2x - 1| - |2x + 1|}{x} = \underline{\textcircled{11}\textcircled{12}}$ .

**Ans:** -4

(b) (5 pts.)  $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 2x - 3} = \underline{\frac{\textcircled{13}}{\textcircled{14}}}$ . (Reduce the answer to the simplest term.)

**Ans:**  $\frac{3}{2}$

(c) (5 pts.)  $\lim_{h \rightarrow 0} \frac{\cos(\pi + h) + 1}{h} = \underline{\textcircled{15}}$ .

**Ans:** 0

(d) (5 pts.) If  $h(2) = 4$  and  $h'(2) = -3$ , find  $\left. \frac{d}{dx} \left( \frac{h(x)}{x} \right) \right|_{x=2} = \underline{\frac{\textcircled{16}\textcircled{17}}{\textcircled{18}}}$ . (Reduce the answer to the simplest term.)

**Ans:**  $\frac{-5}{2}$

(e) Let  $f(x) = \begin{cases} x^2 + b, & \text{if } x \leq 1 \\ ax + 2, & \text{if } x > 1. \end{cases}$

(i) (5 pts.) If  $f(x)$  is differentiable at  $x = 1$ , find  $(a, b) = \underline{(\textcircled{19}, \textcircled{20})}$ .

**Ans:** (2, 3)

(ii) (5 pts.) Continue (i), find  $f'(1) = \underline{\textcircled{21}}$ .

**Ans:** 2

(f) (5 pts.) Set  $f(x) = \begin{cases} cx^2 + 2x, & \text{if } x < 2, \\ x^3 - cx, & \text{if } x \geq 2. \end{cases}$  For what value of the constant  $c$  is the function  $f$  continuous  $(-\infty, \infty)$ ?  $c = \underline{\frac{\textcircled{22}}{\textcircled{23}}}$ . (Reduce the answer to the simplest term.)

**Ans:**  $\frac{2}{3}$

(g) The equation of circle is  $x^2 + y^2 = \text{25}$  5.

(i) (5 pts.) Find the tangent line equation  $y = ax + b$  at  $(-2, 1)$ .  $(a, b) = \underline{(\textcircled{24}, \textcircled{25})}$ .

**Ans:** (2, 5)

(ii) (5 pts.) Find the second derivatives  $y''$  at  $(-2, 1)$ . Answer:  $\underline{\textcircled{26} \textcircled{27}}$ .

**Ans:** -5

(h) (5 pts.) If  $g(x) + x \sin g(x) = x^2$ , find  $g'(0) = \underline{\textcircled{28}}$ .

**Ans:** 0