Calculus (Type A. 2 Pages)

Part I: Multiple-answer questions. Taking a correct answer, you will get 2 points. To select all of the correct answers for each question, you will get one more extra point.

1. (7 pts.) Which of the following statements are correct?

   Ⓐ If a_k is decreasing and $\lim_{k\to\infty} a_k = 0$, then $\sum_{k=1}^{\infty} a_k$ converges;

   Ⓑ If $\lim_{k\to\infty} a_k = a$, then $\sum_{k=1}^{\infty}(a_k - a_{k+2}) = a_1 + a_2 - 2a$;

   Ⓒ Suppose that $a_k = f(k)$ for some continuous function $f : [1,\infty) \to [0,\infty)$ which satisfies $\lim_{x\to\infty} f(x) = 0$. If $\sum_{k=1}^{\infty} a_k$ converges, then $\int_1^{\infty} f(x)\,dx$ converges.

   Ans: Ⓑ

2. (7 pts.) Which of the following series are divergent?

   Ⓐ $\sum_{n=1}^{\infty}(\frac{1}{n} - \sin\frac{1}{n})$

   Ⓑ $\sum_{n=1}^{\infty} \tan^{-1} n$

   Ⓒ $\sum_{n=1}^{\infty} \ln(1+\frac{1}{n})$

   Ans: Ⓑ Ⓒ

3. (7 pts.) Which of the following improper integrals are covergent?

   Ⓐ $\int_1^{\infty} \frac{dx}{\sqrt[3]{x^4+x^3+1}}$

   Ⓑ $\int_0^{1} \frac{dx}{\sqrt{x^5+x^3}}$

   Ⓒ $\int_1^{\infty} \frac{\ln x}{x}\,dx$

   Ans: Ⓐ

4. (7 pts) Which of the following statements are correct?

   Ⓐ Two different level curves of the graph of $z = f(x,y)$ can not intersect;

   Ⓑ The equations for a sphere is a function of three variables;

   Ⓒ The function $f(x,y)$ has both partial derivatives, then it is differentiable.

   Ans: Ⓐ

5. (7 pts) Which of the following statements are correct?

Ⓐ If the power series $\sum_{n=1}^{\infty} a_n x^n$ converges for $x = 3$, then it also converges for $x = -3$;

Ⓑ If $f(x)$ is an odd function, then its nth Maclaurin polynomial contains only terms with odd powers of x;

Ⓒ It is impossible to find a power series whose interval of convergence is $[0, \infty)$.

Ans: Ⓑ Ⓒ

Part II: Fill in the Blanks.

(a) (8 pts.) Let $f(x) = \sin(x^2)$ for all $x \in \mathbb{R}$. Evaluate $f^{(12)}(0) =$ ① and $f^{(26)}(0) =$ $\frac{②③!}{④⑤!}$

Ans: $f^{(12)}(0) = 0, f^{(26)}(0) = \frac{26!}{13!}$

(b) (6 pts.) The radius of convergence of the series $\sum_{n=1}^{\infty} \sin^{-1}(\frac{1}{n^2}) x^n$ is ⑥.

Ans: 1

(c) (5 pts.) Find $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2} =$ ⑦. (If the limit does not exist, please fill "9" in the Blank.)

Ans: 9

(d) (7 pts.) The area of the region bounded by the graph of $f(x) = x\sqrt{4 - x^2}$ and the line $y = 0$ is $\frac{⑧⑨}{⑩}$. (Reduce the answer to the simplest term.)

Ans: $\frac{16}{3}$

(e) (7 pts.) The function $f(x) = 4 - \frac{x^2}{4}$ on the interval $[0, 4]$ is revolved about the line $y = 1$. The volume of the resulting solid is ⑪⑫.⑬π.

Ans: 16.8π

(f) (7 pts) $\int x^2 e^{-x/2}\, dx = -2e^{-x/2}($ ⑭$x^2 +$ ⑮$x +$ ⑯ $) + C$

Ans: $-2e^{-x/2}(x^2 + 4x + 8) + C$

(g) (6 pts) A solid whose base is bounded by the circle $x^2 + y^2 = 9$ with the cross sections perpendicular to the x-axis are equilateral triangles (正三角形). The volume of this solid is ⑰⑱$\sqrt{⑲}$.

Ans: $36\sqrt{3}$

(h) (7 pts.) Let C be the curve of equation $y = 2\sqrt{x}$ for $3 \le x \le 8$. The area of the surface generated by revolving C about the x-axis is $\frac{⑳㉑㉒}{㉓}\pi$. (Reduce the answer to the simplest term.)

Ans: $\frac{152}{3}\pi$

(i) (6 pts) The minimum distance from the point $(-3, 1, 0)$ to the surface $z = \sqrt{2 - 2x - 2y}$ is $\underline{\quad (24) \quad}$

Ans: 2

(j) (6 pts) If $\int_c^x f(t)\,dt = x^2 + x - 2$, the values of c are $\underline{\quad (25) \quad}$ or $\underline{\quad (26)(27) \quad}$

Ans: $-2, 1$