

Calculus (Type B. 2 Pages)

Part I: Choice. Select only one answer choice from a list of four choices.

(42%)

1. $\lim_{h \rightarrow 0^-} (1 - 2h)^{\frac{1}{2h}} =$
(A) e^{-4} (B) e^{-1} (C) e (D) e^4

Ans: (B)

2. The integral $\int_1^{\infty} \frac{1}{x^2} dx$ is
(A) $\frac{1}{2}$ (B) 2 (C) 1 (D) divergent

Ans: (C)

3. $\int_0^1 x e^x dx =$
(A) 1 (B) e (C) $\frac{1}{2}e$ (D) e^2

Ans: (A)

4. $\int_0^3 \sqrt{9 - x^2} dx =$
(A) $\frac{9}{4}\pi$ (B) 3π (C) $\frac{9}{2}\pi$ (D) 9π

Ans: (A)

5. $\int_e^{e^2} \frac{1}{x \ln x} dx =$
(A) 1 (B) $\ln 2$ (C) $e^{\frac{1}{2}}$ (D) $\sqrt{2}$

Ans: (B)

6. The volume of the solid generated by rotating the region bounded by $y = \sqrt{x}$, $x = 1$, $x = 4$, $y = 0$ about the x -axis is

(A) $\frac{9}{2}\pi$ (B) $\frac{4}{3}\pi$ (C) $\frac{19}{6}\pi$ (D) $\frac{15}{2}\pi$

Ans: (D)

Part II: Multiple-answer questions. For these questions, your answer should be one or more. To score all the points for each question, you must select ALL of the correct answers and NONE of the incorrect answers. Missing a correct answer or taking an incorrect one, you will lose 2 points. In the other cases, you will get zero. (10%)

1. Let $f(x) = x^{1/3}(x + 4)$. Which of the followings are **True**?

- (A) f has a local maximum value at $x = 0$.
- (B) f has a local minimum value at $x = -1$.
- (C) $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
- (D) f has an inflection point at $x = 2$.
- (E) f has no absolute minimum value.

Ans: (B)(D)

2. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \sqrt{1 + \frac{3i}{n}} =$

- (A) $\int_1^4 \sqrt{x} dx$
- (B) $\int_0^1 \sqrt{1 + 3x} dx$
- (C) $\int_0^3 \sqrt{1 + x} dx$
- (D) $\frac{14}{9}$
- (E) $\frac{14}{3}$

Ans: (A)(C)(E)

Part III: Fill in the Blanks. (48%)

(a) Find the limit $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \frac{\textcircled{1}}{\textcircled{2}}$. Ans: $\frac{1}{2}$

(b) Let $f(x) = \int_0^x x \cos(t^2) dt$, find $f''(0) = \textcircled{3}$. Ans: 2

(c) Find $\frac{dy}{dx}$ at the point (1, 1) if $\ln y + xy = 1$. Answer: $\frac{\textcircled{4}\textcircled{5}}{\textcircled{6}}$. Ans: $-\frac{1}{2}$

(d) Let $f'(x) = \frac{(x+1)^2}{x}$, $x \neq 0$, $f(1) = 2$, find $f(-1) = \textcircled{7}\textcircled{8}$. Ans: -2

(e) Let $f(x) = \frac{(2x+1)^2 \sqrt{x^2+1}}{(x^3-3x+1)^3}$, find $f'(0) = \textcircled{9}\textcircled{10}$. Ans: 13

(f) Find the area enclosed by the line $y = x - 2$ and the parabola $y^2 = x$. Answer: $\frac{\textcircled{11}}{\textcircled{12}}$.

Ans: $\frac{9}{2}$