Calculus Exam (Group B)

Part I: Multiple answer questions. To get all points for each question, you must select ALL correct choices and NONE of incorrect choices. If you miss a correct choice or taking an incorrect choice, then you will lose 50% of the full points. For all other cases you will get zero points.

Problem 1. (5 points) Let

$$f(x) = \frac{x^2 + x - 6}{|x - 2|}.$$

Which of the following statements are correct?

(A) $\lim_{x \to 2+} f(x) = 5$

(B) $\lim_{x \to 2-} f(x) = 5$

(C) $\lim_{x \to 2-} f(x) = -5$

(D) $\lim_{x \to 2} f(x) = 5$

Answer: (A), (C)

Problem 2. (5 points) Let $f(x) = x(3x - 4)^{2/3}$. Which of the following statements are correct?

(A) $x = \frac{8}{5}$ is an inflection point.

(B) $f(\frac{4}{5})$ is a local minimum.

(C) $x = \frac{4}{3}$ is an inflection point.

(D) $f(\frac{4}{3})$ is a local minimum.

Answer: (A), (D)   (From online test system)

Problem 3. (5 points) If $f(2) = 4$, $f(4) = 7$, $f'(2) = 2$, $f'(4) = 1$, $f'(8) = -3$, let

$g(x) = f(f(x))$, $h(x) = f(x^3)$, $k(x) = [f(x)]^3$. Which of the following statements are correct?

(A) $g'(2) = 3$

(B) $g'(2) = 2$

(C) $h'(2) = 5$

(D) $k'(2) = 96$

Answer: (B), (D) (From online test system)

Problem 4. (5 points) Assume

$$f(x) = \begin{cases} 3\sin x + 2\cos x & x < 0 \\ ax + b & 0 \le x < 2 \\ \sqrt{x+2} & x \ge 2 \end{cases}$$

is continuous at $x = 0, 2$. Which of the following are true?

(A) $a + b = 1$

(B) $a + b = 2$

(C) $ab = 1$

(D) $b = 2$

Answer: (B), (D)

Problem 5. (6 points) Assume

$$f(x) = \begin{cases} ax^2 & x \le -1 \\ x^2 + bx & x > -1 \end{cases}$$

is differentiable at $x = -1$. Which of the following are true?

(A) $a + b = 1$

(B) $a + b = 2$

(C) $a + 2b = 1$

(D) $2a - b = 2$

Answer: (A), (C), (D)

Problem 6. (4 points) Let (a,b) be the largest interval in which $f(x)=x^3-3x^2+2$ is decreasing.

Which of the following are true?

(A) $a+b=0$

(B) $b-a=2$

(C) $ab=0$

(D) $ab=-1$

Answer: (B), (C)

Problem 7. (5 points) Let $f(x)=\sin x-\cos x$ be defined on $[-\pi,3\pi]$. By the Mean Value Theorem, there is at least one $c\in(-\pi,3\pi)$ such that the tangent line to $f(x)$ at $x=c$ is parallel to the secant line through the endpoints $(-\pi,f(-\pi))$, $(3\pi,f(3\pi))$. Which of the following values of c satisfy the conclusion to the Mean Value Theorem?

(A) $-\dfrac{\pi}{4}$

(B) $\dfrac{3}{4}\pi$

(C) $\dfrac{5}{4}\pi$

(D) $\dfrac{7}{4}\pi$

Answer: (A), (B), (D)

Part II: Single answer questions. Select only ONE correct choice from a list of four choices.

Problem 1. (5 points) Find numbers a and b such that $\lim\limits_{x\to0}\dfrac{\sqrt{ax+b}-2}{x}=1$.

(A) $(a,b)=(2,2)$

(B) $(a,b)=(2,4)$

(C) $(a,b)=(4,4)$

(D) $(a,b)=(4,2)$

Answer: (C)

Problem 2. (5 points) If $h(x) = \dfrac{x+1}{x-5}$, then $\displaystyle\lim_{x \to 5^-} h(x) =$

(A) 1

(B) $-\dfrac{1}{5}$

(C) ∞

(D) $-\infty$

Answer: (D)

Problem 3. (5 points) If $f(t) = \sqrt{4t+1}$, then $f''(2) =$

(A) $-\dfrac{1}{27}$

(B) $-\dfrac{1}{108}$

(C) $-\dfrac{4}{108}$

(D) $-\dfrac{4}{27}$

Answer: (D)

Problem 4. (5 points) If $g(x) + x\sin g(x) = x^2$, then $g'(0) =$

(A) 2

(B) 1

(C) $\dfrac{1}{2}$

(D) 0

Answer: (D)

Problem 5. (5 points) Let $f(x) = 2x^3 + x^2 + 2x$ for all $x \in \mathbb{R}$. How many critical numbers does f have?

(A) 0

(B) 1

(C) 2

(D) 3

Answer: (A)

Problem 6. (5 points) Let $f(t) = 2\cos t + \sin 2t$. The absolute maximum value of f on $[0, \frac{\pi}{2}]$ is

(A) $\dfrac{3\sqrt{3}}{2}$

(B) 2

(C) $\sqrt{3}$

(D) 0

Answer: (A)

Problem 7. (5 points) The slope of the tangent to the curve $y = \sqrt{x} / (1 + x^2)$ at the point $(1, \frac{1}{2})$ is

(A) $-\dfrac{3}{8}$

(B) $-\dfrac{1}{4}$

(C) $-\dfrac{1}{8}$

(D) $-\dfrac{3}{4}$

Answer: (B)

Problem 8. (6 points) Find $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$.

(A) 3

(B) -3

(C) 0

(D) It does not exist.

Answer: (B)

(Section 1.2 online test system)

Problem 9. (6 points) Assume $\displaystyle\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists. Find the value of a.

(A) 15

(B) 12

(C) 9

(D) 6

Answer: (A)

(Section 1.2 online test system)

Problem 10. (6 points) Find $\displaystyle\lim_{x \to 1} \frac{\sin(x-1)}{x^3 + x - 2}$.

(A) 0

(B) $\dfrac{1}{2}$

(C) $\dfrac{1}{4}$

(D) $\dfrac{1}{3}$

Answer: (C)

Problem 11. (6 points) In which of the following interval or intervals is $f(x) = x^4 - 4x^3 + 12$ concave

upward?

(A) $(-\infty, 1), (2, \infty)$

(B) $(-\infty, 0), (2, \infty)$

(C) $(1, 2)$

(D) $(0, 2)$

Answer: (B)

Problem 12 (6 points) $x^2 + y^2 = 20$ is an equation of circle. Find $y''\big|_{(-4,2)}$.

(A) -5

(B) $-\dfrac{5}{2}$

(C) -2

(D) $\dfrac{-5}{16}$

Answer: (B)