

Calculus Exam (Group B, C)

Part I: Multiple answer questions.

To get all points for each question, you must select ALL correct choices and NONE of incorrect choices. If you miss a correct choice or taking an incorrect choice, then you will lose 50% of the full points. For all other cases you will get zero points.

Which of the following statements are correct?

Problem 1. (5 points) If $s''(t) = 6t - 4$; $s(0) = -2$, $s'(0) = 4$ then

(A) $s'(1) = 3$ (B) $s'(-1) = 7$ (C) $s(1) = 1$ (D) $s(-1) = -7$.

Ans: (A), (C).

Problem 2. (5 points) Assume $A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left[1 - \left(1 + \frac{3i}{n} \right)^2 \right]$. Which of the following results are true?

(A) $A = \int_0^3 (1 - x^2) dx$ (B) $A = \int_1^4 (1 - x^2) dx$ (C) -6 (D) -18

Ans: (B), (D).

(question from online test system).

Problem 3. (6 points) Let $G(x) = \int_{-x^2}^{x^2} \frac{t^2}{1+t^2} dt$. Which of the following results are true?

(A) $G'(0) = 0$ (B) $G'(1) = 2$ (C) $G'(-1) = -2$ (D) $G'(2) = 1$

Ans: (A), (B), (C).

Problem 4. (6 points) Assume $A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{2}{c_i^2} \right) \Delta x_i$ is the limit of Riemann sum over the

interval $[a, b]$. Which of the following results are true?

- (A) $A = 1$ over $[1, 2]$ (B) $A = \frac{4}{3}$ over $[1, 3]$ (C) $A = \frac{3}{2}$ over $[1, 4]$ (D) $A = \frac{8}{5}$ over $[1, 5]$

Ans: (A), (B), (C), (D).

(question from online test system).

Problem 5. (6 points) Let $\int_0^\pi e^x \sin x \, dx = a + be^\pi$. Which of the following results are true?

- (A) $a + b = 1$ (B) $b = \frac{1}{2}$ (C) $a - b = 1$ (D) $a + b = 2$

Ans: (A), (B).

Problem 6. (6 points) Let $\int_0^4 \frac{\sqrt{x}}{1+\sqrt{x}} \, dx = a + 2\ln b$. Which of the following results are true?

- (A) $a = 0$ (B) $a = 1$ (C) $b = 2$ (D) $b = 3$.

Ans: (A), (D).

Problem 7. (6 points) At which of the following values of n the improper integral $\int_1^\infty \frac{1}{x^n} \, dx$ converges?

- (A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2.

Ans: (C), (D).

Part II: Single answer questions.

Select only ONE correct choice from a list of four choices.

Problem 1. (5 points) Find $\int 2\sin x \cos(\cos x) \, dx$.

- (A) $-\frac{1}{2}\cos(\cos 2x) + C$ (B) $-2\cos(\sin x) + C$ (C) $-2\sin(\cos x) + C$ (D) $-2\cos(\cos x) + C$

Ans: (C).

Problem 2. (5 points) Find $\int \frac{1+\sqrt{x}+3x}{\sqrt{x}} dx$.

(A) $x + \frac{3}{2}x^2 + C$ (B) $2x^{3/2} + x + 2\sqrt{x} + C$ (C) $\frac{9}{4}\sqrt{x} + 1 + \frac{3}{2}x^{-1/2} + C$ (D) $\frac{3}{2}x^2 + \frac{2}{3}x^{3/2} + x - 2\sqrt{x} + C$

Ans: (B).

Problem 3. (5 points) Find $\int_{-2}^2 (2 + x\sqrt{1+x^2+x^4}) dx$.

(A) 5 (B) 6 (C) 7 (D) 8

Ans: (D).

(From online test system)

Problem 4. (5 points) Find $\int_0^{\pi/4} \tan^3 x dx$.

(A) $\frac{1}{2} + \ln \frac{1}{\sqrt{2}}$ (B) $\frac{1}{2} - \ln \frac{1}{\sqrt{2}}$ (C) $\frac{1}{2} + \ln \frac{1}{2}$ (D) $\frac{1}{2} - \ln \frac{1}{2}$

Ans: (A).

(From online test system)

Problem 5. (5 points) Find $\int_0^5 \frac{1}{\sqrt{25-x^2}} dx$.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π

Ans: (C).

(From online test system)

Problem 6. (5 points) Find $\int_0^1 x^2 e^{-x} dx$.

(A) $1 + \frac{3}{e}$ (B) $2 + \frac{5}{e}$ (C) $1 - \frac{3}{e}$ (D) $2 - \frac{5}{e}$

Ans: (D).

Problem 7. (5 points) Assume $f(x) = (x^2 + 1)^{\sin x}$. Find $f'(0)$.

(A) -1 (B) 0 (C) 1 (D) 2

Ans: (B).

(from online test system)

Problem 8. (5 points) Find $\int_0^1 x^2 + 2^x dx$.

(A) $\frac{1}{3} + \frac{1}{\ln 2}$ (B) $\frac{1}{3} + \frac{1}{\ln 3}$ (C) $\frac{1}{3} + \frac{2}{\ln 2}$ (D) $\frac{1}{3} + \frac{2}{\ln 3}$

Ans: (A).

Problem 9. (5 points) Find $\lim_{x \rightarrow \infty} e^{-2x} \int_0^x \frac{e^t}{1+e^t} dt$.

(A) 0 (B) $\frac{1}{2}$ (C) 2 (D) It does not exist.

Ans: (A).

Problem 10. (5 points) Find the area of the region bounded by the graphs of $f(x) = \ln x$, $x = 0$, $x = 1$ and the x -axis.

(A) 2 (B) $\ln 2$ (C) 1 (D) It does not exist.

Ans: (C).

Problem 11. (5 points) Assume $f(x) = x^2$, $g(x) = \sqrt{x}$. Find the area of region bounded by the graphs of $f(x)$, $g(x)$.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 2.

Ans: (B).

Problem 12. (5 points) Find the volume of the solid generated by revolving the region bound by the graphs of $f(x)$, $g(x)$ in problem 11 about the x -axis.

(A) $\frac{3}{20}\pi$ (B) $\frac{3}{10}\pi$ (C) $\frac{3}{5}\pi$ (D) $\frac{3}{4}\pi$.

Ans: (B).