Calculus Exam (Group A)

I Multiple Answer Questions

To get all points for each question, you must select ALL correct choices and NONE of incorrect choices. If you miss a correct choice or taking an incorrect choice, then you will lose 50% of the full points. For all other cases you will get zero points.

Problem 1. (7 points) *Let a_0, a_1, a_2 be real numbers such that the limit*

$$\lim_{x\to 0} \frac{\ln(1+x) - (a_0 + a_1 x + a_2 x^2)}{x^3}$$

exists. Which of the following statements are correct?

(A) $a_0 = 0$.

(B) $a_1 = 1$.

(C) $a_2 = \frac{1}{2}$.

(D) *The limit is zero.*

Answer: (A), (B). Modified from the Online Test System §**10.2**.

Problem 2. (7 points) *Which of the following statements are correct?*

(A) $\int_{-\pi}^{\pi} \sin(x)\,\mathrm{d}x = \int_{-\pi}^{\pi} \cos(x)\,\mathrm{d}x$.

(B) $\int_{-\pi}^{\pi} \sin^2(x)\,\mathrm{d}x = \int_{-\pi}^{\pi} \cos^2(x)\,\mathrm{d}x$.

(C) $\int_{-\pi}^{\pi} \sin^4(x)\,\mathrm{d}x = \int_{-\pi}^{\pi} \cos^4(x)\,\mathrm{d}x$.

(D) $\int_{-\pi}^{\pi} \sin(x)\cos(x)\,\mathrm{d}x = 0$.

Answer: (B), (C), (D).

Problem 3. (8 points) *Consider the parametric curve*

$$\Gamma = \{(x, y) : x = t - \ln t,\ y = t + \ln t,\ \textit{for } t > 0\}.$$

Which of the following statements are correct?

(A) $(1, 1) \in \Gamma$.

(B) Γ *is bounded, that is, it is contained in the interior of some circle.*

(C) *The tangent line of* Γ *passing through the point* $(2 - \ln 2, 2 + \ln 2)$ *has slope* 3.

(D) Γ *is concave upward.*

Answer: (A), (C). Modified from the Textbook §**10.2** P. 19.

Problem 4. (8 points) *Let* $a_n = \sqrt[n]{2} - 1$ *for* $n \in \mathbb{N}$. *Which of the following statements are correct?*

(A) $\lim_{n\to\infty} a_n = 0$.

(B) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$.

(C) $\lim_{n\to\infty} \sqrt[n]{a_n} = 0$.

(D) $\sum_{n=1}^{\infty} a_n$ *diverges.*

Answer: (A), (B), (D). Modified from the Textbook §**11.7** P. 48.

Problem 5. (9 points) *Consider the series* $s = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{8^n}$ *and its partial sum* $s_k = \sum_{n=1}^{k}(-1)^{n-1}\frac{n}{8^n}$ *for* $k \in \mathbb{N}$. *Which of the following statements are correct?*

(A) $\lim_{k\to\infty} s_k \neq s$, *that is,* s *diverges.*

(B) $|s - s_k| < 10^{-3}$ *for all* $k \geq 3$.

(C) $s = \frac{1}{9}$.

(D) $10^{-3} < |s - s_k|$ *for* $k = 1, 2$.

Answer: (B), (D).

Problem 6. (9 points) *Consider* $f(x) = e^{-x^2}\cos(x)$. *Which of the following statements are correct? Notation:* $f^{(k)}(0)$ *denotes the* k*-th derivative of* f *at* $x = 0$, *for instance,* $f^{(2)}(0) = f''(0)$.

(A) *The sequence* $\{f(n)\}_{n\in\mathbb{N}}$ *converges.*

(B) $\sum_{n=0}^{\infty} f(n)$ *diverges.*

(C) $f^{(4)}(0) = 3$.

(D) $f^{(7)}(0) = 0$.

Answer: (A), (D). Modified from the Textbook §**11.10** P. 73.

II Single Answer Questions

Select only ONE correct choice from a list of four choices.

Problem 1. (5 points) *What is* $f'(1)$, *where*

$$f(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,\mathrm{d}z?$$

(A) 1.

(B) $\frac{1}{2}$.

(C) $\frac{1}{3}$.

(D) $\frac{1}{4}$.

Answer: (D). From the Textbook §**4.3** P. 16.

Problem 2. (5 points) *Consider* $f(x) = x^3 - |x|$. *What is the area of the region enclosed by* $x = -1$, *the graph of* f, $x = 2$, *and* $y = 0$*?*

(A) -1.

(B) 1.

(C) $\frac{13}{4}$.

(D) $\frac{17}{5}$.

Answer: (C).

Problem 3. (6 points) *What is the volume of the solid obtained by rotating the region bounded by* $y = x^2$ *and* $y = \sqrt{x}$ *around the* x*-axis?*

(A) $\frac{3}{4}\pi$.

(B) $\frac{3}{8}\pi$.

(C) $\frac{3}{10}\pi$.

(D) $\frac{3}{14}\pi$.

Answer: (C).

Problem 4. (6 points) *What is* $\lim_{x\to 0}(\csc(x) - \cot(x))$*?*

(A) -1.

(B) 0.

(C) 1.

(D) ∞.

Answer: (B). From the Textbook §**6.8** P. 52.

Problem 5. (6 points) *Let* a, b, c *be real numbers such that the partial fraction*

$$\frac{1}{x^2(x-1)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x-1}$$

holds for all $x \in \mathbb{R} \setminus \{0, 1\}$. *Then* $a + b + c =$

(A) -1.

(B) 0.

(C) 1.

(D) 2.

Answer: (A).

Problem 6. (6 points) *What is the definite integral*

$$\int_0^1 \sqrt{x}\, e^{\sqrt{x}}\, \mathrm{d}x?$$

(A) $\sqrt{e}$.

(B) e.

(C) $2e - 4$.

(D) $4e - 8$.

Answer: (C). Modified from the Textbook §**7.5** P. 71.

Problem 7. (6 points) *With which value p does the following improper integral converges:*

$$\int_e^\infty \frac{1}{x(\ln x)^p}\, \mathrm{d}x?$$

(A) 0.

(B) $\frac{1}{2}$.

(C) 1.

(D) $\frac{3}{2}$.

Answer: (D). From the Online Test System §**7.5**.

Problem 8. (6 points) *What is the arc length of the cardioid given by $r = 1 + \cos(\vartheta)$ for $\vartheta \in [0, 2\pi]$?*

(A) e.

(B) π.

(C) 8.

(D) 16.

Answer: (C). From the Online Test System §**9.1**.

Problem 9. (6 points) *Consider the series*

$$s = \sum_{n=0}^{\infty} (-1)^n = 1 - 1 + 1 - 1 + 1 - \cdots .$$

Which of the following statements are correct?

(A) $s = 0$ *converges, because* $s = (1 - 1) + (1 - 1) + \cdots = 0 + 0 + \cdots = 0$.

(B) $s = 1$ *converges, because* $s = 1 + (-1 + 1) + (-1 + 1) + \cdots = 1 + 0 + 0 + \cdots = 1$.

(C) $s = \frac{1}{2}$ *converges, because both* $s = 0$ *and* $s = 1$ *imply* $s + s = 0 + 1 = 1$.

(D) s *diverges.*

Answer: (D).