Calculus Exam (Group A)

I Multiple Answer Questions (複選題)

To get all points for each question, you must select ALL correct choices and NONE of incorrect choices. If you miss a correct choice or taking an incorrect choice, then you will lose 50% of the full points. For all other cases you will get zero points.

Problem 1. (8 points) *A function f is called* ***even*** *if $f(-x) = f(x)$ for all x in its domain, and* ***odd*** *if $f(-x) = -f(x)$ for all such x. Which of the following statements are correct?*

(A) *If f and g are both odd functions, then fg is also an odd function.*

(B) *If f is an even function and g is an odd function, then fg is an odd function.*

(C) *Let f be a function with domain $\mathbb{R}$. Then $f(x) - f(-x)$ is an odd function.*

(D) *The derivative function of an even function is an odd function.*

Answer: (B), (C), (D). From the Textbook §1.1 Exercise 88, §1.3 Exercise 71, and §2.2 Exercise 61.

Problem 2. (8 points) *Let*

$$f(x) = \begin{cases} x\sin(\frac{1}{x}), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Which of the following statements are correct?

(A) $\lim_{x\to 0} f(x) = 0$.

(B) *f is continuous at* 0.

(C) *f is differentiable at* 0.

(D) $\lim_{x\to\infty} f(x) = 1$.

Answer: (A), (B), (D). From the Textbook §2.1 Exercise 57.

Problem 3. (8 points) *Let f be a differentiable function satisfying $f(1) = 10$ and $3 \leq f'(x) \leq 5$ for all $1 \leq x \leq 5$. What can $f(4)$ possibly be?*

(A) 18.

(B) 21.

(C) 24.

(D) 25.

Answer: (B), (C), (D) From the Textbook §3.2 Exercise 29.

Problem 4. (8 points) *Consider the function* $f(t) = 2\cos(t) + \sin(2t)$ *defined on the interval* $[0, \frac{\pi}{2}]$. *Which of the following statements are correct?*

(A) $\frac{3\sqrt{3}}{2}$ *is a local maximum.*

(B) $\frac{3\sqrt{3}}{2}$ *is the absolute maximum.*

(C) 0 *is a local minimum.*

(D) 0 *is the absolute minimum.*

Answer: (A), (B), (D). From the Textbook §3.1 Exercise 59.

Problem 5. (8 points) *Which of the following statements about the curve*

$$y = f(x) = \frac{2x^2}{x^2 - 1}$$

are correct?

(A) f *is an even function and so the curve is symmetric with respect to the* y*-axis.*

(B) *There are exactly two asymptotes of the curve.*

(C) *The only critical point is* $x = 0$ *and* f *is strictly increasing on* $(-\infty, -1) \cup (-1, 0)$.

(D) f *is concave upward on the interval* $(-1, 1)$.

Answer: (A), (C).

II Single Answer Questions (單選題)

Select only ONE correct choice from a list of four choices.

Problem 1. (6 points) *Let* $f(x) = 4x + 2$. *We have* $\lim_{x\to 1} f(x) = 6$. *By the definition of limit, for any* $\epsilon > 0$, *we can always find a* $\delta > 0$ *such that if* $0 < |x - 1| < \delta$, *then* $|f(x) - 6| < \epsilon$. *Which one of the following pairs of* (ϵ, δ) *works?*

(A) $(\epsilon, \delta) = (1, \frac{1}{2})$.

(B) $(\epsilon, \delta) = (\frac{1}{2}, \frac{1}{4})$.

(C) $(\epsilon, \delta) = (\frac{1}{3}, \frac{1}{8})$.

(D) $(\epsilon, \delta) = (\frac{1}{4}, \frac{1}{16})$.

Answer: (D). From the Online Test System §1.2.

Problem 2. (6 points) *Which one of the following statement is correct?*

(A) $\lim_{x\to 4^-} \frac{x-4}{\sqrt{x}-2} = -4$.

(B) $\lim_{x\to 3^+} \frac{|x-3|}{x-3} = -1$.

(C) $\lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right) = 0$.

(D) $\lim_{x\to 0} \sin(x + \pi\cos(x)) = \sin(\pi\cos(0))$.

Answer: (D). From the Online Test System §1.2.

Problem 3. (6 points) *Consider the curve* $\{(x,y) : x^2y^2 + xy = 2\}$. *How many statements below are correct?*

(i) *The curve is the graph of a function* y *of* x.

(ii) *There are two points on the curve where the slope of the tangent line is* -1.

(iii) *The line defined by* $y - 2x + 4 = 0$ *is the tangent line to the curve at* $(1, -2)$.

(A) *Three.*

(B) *Two.*

(C) *One.*

(D) *Zero.*

Answer: (B). Only (ii) and (iii) are true.

Problem 4. (6 points) *Consider the function* $y = f(x) = |\sin(x)|$. *How many statements below are correct?*

(i) *The derivative of* y *at* $x = \frac{\pi}{2}$ *is zero.*

(ii) *The derivative function* y' *is defined for all* $x \neq k\pi$, *where* k *is any integer.*

(iii) *If* y *represents a position function and* x *represents time, then* $\sin(1)$ *is the instantaneous acceleration at time* $x = 1$.

(A) *Three.*

(B) *Two.*

(C) *One.*

(D) *Zero.*

Answer: (B). Only (i) and (ii) are true.

Problem 5. (6 points) *How many statements below are correct?*

(i) *If* $\lim_{x\to a} g(x) = b$, $\lim_{y\to b} f(y) = c$, *where* $a, b, c \in \mathbb{R}$ *and* $f(b)$ *is defined, then* $\lim_{x\to a} f(g(x)) = f(b)$.

(ii) *If* g *is continuous at* x *and* f *is continuous at* $g(x)$, *then the composition* $f \circ g$ *is continuous at* x.

(iii) *If* g *is differentiable at* x *and* f *is differentiable at* $g(x)$, *then* $f \circ g$ *is differentiable at* x.

(A) *Three.*

(B) *Two.*

(C) *One.*

(D) *Zero.*

Answer: (B). Only (ii) and (iii) are true.

Problem 6. (6 points) *Suppose* $g(1000) = 10$ *and* $g'(x) = \frac{1}{3}x^{-2/3}$ *for all* $x > 0$. *What is the linearization* L *of* g *at* $x = 1000$*?*

(A) $L(x) = 10 + \frac{1}{300}(x - 1000)$.

(B) $L(x) = 10 + \frac{1}{400}(x - 1000)$.

(C) $L(x) = 10 + \frac{1}{500}(x - 1000)$.

(D) $L(x) = 10 + \frac{1}{600}(x - 1000)$.

Answer: (A). From the Textbook §2.9 Exercise 33.

Problem 7. (6 points) *Use the linearization* L *of* g *at* $x = 1000$ *as above to estimate* $g(1001) \approx$*?*

(A) $\frac{3001}{300}$.

(B) $\frac{4001}{400}$.

(C) $\frac{5001}{500}$.

(D) $\frac{6001}{600}$.

Answer: (A). From the Textbook §2.9 Exercise 33.

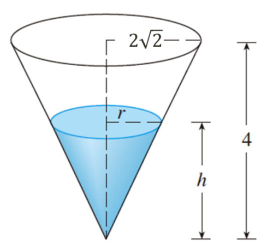
Problem 8. (6 points) *A water tank has the shape of an inverted circular cone with base radius* $2\sqrt{2}$m *and height* 4m. *If water is being pumped into the tank at a rate of* $3\text{m}^3/\text{min}$, *find the rate at which the water level is rising, i.e.,* $\frac{dh}{dt}$, *when the water is* 3m *deep.*

(A) $\frac{8}{9\pi}$ m/min.

(B) $\frac{2\sqrt{2}}{9\pi}$ m/min.

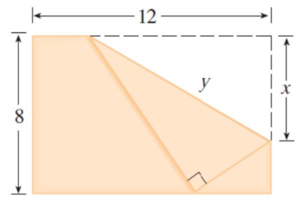
(C) $\frac{2\sqrt{2}}{3\pi}$ m/min.

(D) $\frac{2}{3\pi}$ m/min.



Answer: (D). From the Online Test System §3.1.

Problem 9. (6 points) *The upper right-hand corner of a piece of paper, 12cm by 8cm, as depicted in the figure, is folded over to the bottom edge. Express y as a function of x.*



(A) $y = \sqrt{\frac{x^3}{x-4}}$.

(B) $y = x^2 + \frac{4x^2}{x-4}$.

(C) $y = \sqrt{\frac{x^3}{x-6}}$.

(D) $y = x^2 + \frac{4x^2}{x-6}$.

Answer: (A).

Problem 10. (6 points) *Based on the above setting, how would you fold it so as to minimize the length y of the fold? Find the minimum value $y =$?*

(A) 6.

(B) $6\sqrt{2}$.

(C) $6\sqrt{3}$.

(D) $8\sqrt{2}$.

Answer: (C).