Calculus Exam (Group B、C)

Part I. Multiple answer questions.（複選題）

To get all points for each question, you must select ALL correct choices and NONE of incorrect choices. If you miss a correct choice or taking an incorrect choice, then you will lose 50% of the full points. For all other cases you will get zero points.

Problem 1. (5 points) $f(x)=\begin{cases} x^4\sin\left(\frac{4}{x}\right); x\neq 0 \\ 0\,; x=0 \end{cases}$, Find (a)$f'(0)$ =? (b)$f'(1)$ =?

(A) $f'(0)=1$

(B) $f'(0)=0$

(C) $f'(1)=4\sin(4)-4\cos(4)$

(D) $f'(1)=4\sin(2)-2\cos(2)$

Answer: (B),(C) (From online test system)

Problem 2. (5 points) Let $f(x)=\frac{\pi}{2}-x$ and $g(x)=\frac{2\sin x}{1+\cos x}$. Which of the following are true?

(A) $g(f(0))=2$

(B) $(g\circ f)\left(\frac{\pi}{2}\right)=0$

(C) $f\left(g\left(\frac{\pi}{2}\right)\right)=\frac{\pi}{2}$

(D) $(f\circ f)\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$

Answer: (A),(B),(D)

Problem 3. (5 points) Find all critical numbers for the function $f(x)=(x+3)^4(x-2)^3$.

(A) 0

(B) -3

(C) $-\frac{1}{7}$

(D) 2

Answer: (B),(C),(D)

Problem 4. (5 points) Suppose the derivative of the function $y=f(x)$ is

$f'(x) = (x-1)^2(x-2)(x-4)$. Which statement(s) of the followings is(are) correct?

(A) f has a local maximum at $x = 2$.
(B) f has a local minimum at $x = 4$.
(C) f has a point of inflection at $x = 1$.
(D) f has a local maximum at $x = 1$.

Answer: (A) (B) (C)

Problem 5. (5 points) Given $f(x) = \begin{cases} x^2 - 1, & -1 \le x \le 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$, which statement(s) of the followings is (are) NOT correct?

(A) f is continuous at $x = 1$.
(B) f is continuous from the right at $x = -1$.
(C) $\lim_{x \to 3^-} f(x)$ does not exist.
(D) If we redefine $f(1) = 2$, then f can be changed to be continuous at $x = 1$.

Answer: (A)(C)

Problem 6. (5 points) If $f(x) = \frac{2x^2+ax+b}{x^2+cx+d}$ satisfies the followings:

(1) f has vertical asymptotes $x = -1$ and $x = 2$, horizontal asymptote $y = 2$.

(2) $\lim_{x \to 1} f(x) = 0$

(3) $\lim_{x \to 0} f(x) = 3$

(A) $a = 4$
(B) $b = 6$
(C) $c = -1$
(D) $d = -2$

Answer: (A) (C) (D) (From online test system)

Part II. Single answer questions.（單選題）

Select only ONE correct choice from a list of four choices.

Problem 1. (5 points) IF $f(x) = \begin{cases} x+1; x<0 \\ -x^2+1; 0 \le x \le 1 \\ \sin(\pi x); x>1 \end{cases}$, then how many points of discontinuity does $f(x)$ have?

(A) 0

(B) 1

(C) 2

(D) Infinitely many

Answer: (A) (From online test system)

Problem 2. (5 points) If $f(x) = \sin[\cos(x^2)]$, then $f'(x) =$?

(A) $-2x\sin[\cos(x^2)] \cdot \sin(x^2)$

(B) $2x\sin[\cos(x^2)] \cdot \sin(x^2)$

(C) $-2x\sin[\cos(x^2)] \cdot \sin(x^2)$

(D) $-2x\cos[\cos(x^2)] \cdot \sin(x^2)$

Answer: (D) (From online test system)

Problem 3. (5 points) Find the limit $\lim_{x\to 8} \frac{x}{x-8}$, if it exists.

(A) $\frac{1}{8}$

(B) $\frac{8}{7}$

(C) 8

(D) The limit does not exist

Answer: (D)

Problem 4. (5 points) Find the slope of the tangent line to the graph of the function at the given point and determine an equation of the tangent line.

$$f(x) = 2x + 8 \text{ at } (-1,6)$$

(A) $2; y = 2x + 8$

(B) $2; y = 2x$

(C) $-2; y = -2x$

(D) $-2; y = -2x + 8$

Answer: (A)

Problem 5. (5 points) $x^2 + y^2 = 25$, find the value of $\frac{dy}{dx}$ at the point (3, 4).

(A) $-\frac{3}{4}$

(B) 0

(C) 1

(D) $\frac{3}{4}$

Answer: (A)

Problem 6. (5 points) If $f(x) = \sqrt[3]{x^2 - 1}$, find $f''(3)$

(A) $-\frac{3}{4}$

(B) $-\frac{1}{12}$

(C) $\frac{1}{12}$

(D) $\frac{3}{4}$

Answer: (B)

Problem 7. (5 points) On what interval is the function $f(x) = \frac{x}{x^2+1}$ increasing?

(A) [-1,1]

(B) [-2,1]

(C) $(\infty, 2)$

(D) $(-\infty, 1)$

Answer: (A)

Problem 8. (5 points) $\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\right)$ =?

(A) 0.5

(B) –0.5

(C) –1

(D) the limit does not exist

Answer: (B)

Problem 9. (5 points) Let $f(x) = (x-3)^{-2}$. There is no value of c in (1, 4) such that $f(4) - f(1) = f'(c)(4-1)$. Why does this not contradict the Mean Value Theorem?

(A) $f(x) = (x-3)^{-2}$ can not be differentiated in (1, 4).
(B) $f(x) = (x-3)^{-2}$ does not have limit in (1, 4).
(C) $f(x) = (x-3)^{-2}$ is not defined in (1, 4).
(D) $f(x) = (x-3)^{-2}$ is not continuous in (1, 4).

Answer: (D)

Problem 10. (5 points) Find equations for the tangent and normal to $y^2(2-x) = x^3$ at (1, 1).

(A) tangent: $x - 2y = 1$; normal: $2x - y = 1$
(B) tangent: $x + 2y = 3$; normal: $x + 2y = 3$
(C) tangent: $x + 2y = 3$; normal: $2x - y = 1$
(D) tangent: $2x - y = 1$; normal: $x + 2y = 3$

Answer: (D)

Problem 11. (5 points) Only one of these calculations is correct. Which one?

(A) $\lim_{x\to 0} \frac{x^2-2x}{x^2-\sin x} = \lim_{x\to 0} \frac{2x-2}{2x-\cos x} = \lim_{x\to 0} \frac{2}{2+\sin x} = \frac{2}{2+0} = 1$

(B) $\lim_{x\to 0} \frac{x^2-2x}{x^2-\sin x} = \lim_{x\to 0} \frac{2x-2}{2x-\cos x} = \frac{-2}{0-1} = 2$

(C) $\lim_{x\to 3} \frac{x-3}{x^2-3} = \lim_{x\to 0} \frac{1}{2x} = \frac{1}{6}$

(D) $\lim_{x\to(\pi/2)^-} \frac{\sec x}{\tan x} = \frac{-\infty}{\infty} = -1$

Answer: (B)

Problem 12. (5 points) Find a value of c that makes the function below is continuous

at $x = 0$.

$$f(x) = \begin{cases} \dfrac{9x - 3\sin 3x}{5x^3}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

(A) 27/30
(B) –81/30
(C) 81/30
(D) 27/15

Answer: (C)

Problem 13. (5 points) Find the relative maximum of the function with domain $(-\infty, \infty)$.

$f(x) = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x + 4$

A. $8\frac{2}{3}$

B. $8\frac{1}{2}$

C. $9\frac{2}{3}$

D. $9\frac{1}{2}$

Answer: (A) (From online test system)

Problem 14. (5 points) Find the value of the limit $\lim_{x\to\infty}(\sqrt{x^2 + x} - x)$

A. $-\sqrt{2}$
B. $\sqrt{2}$
C. $\frac{1}{2}$
D. ∞

Answer: (C)