

## Calculus Exam (Group B、C)

### Part I. Multiple answer questions. (複選題)

To get all points for each question, you must select ALL correct choices and NO incorrect choices. If you miss a correct choice or choose an additional incorrect choice then you will lose 50% of the full points. In all other cases, you will get zero points.

**Problem 1.** (6 points) Assume  $f''(x) = 12x - 4$  and  $f(x)$  has a maximum value equal 2 at  $x = -1$ . Which of the following are true?

(A)  $f'(0) = -8$  (B)  $f'(1) = -8$  (C)  $f(0) = 3$  (D)  $f(1) = -14$ .

Ans: B, D

**Problem 2.** (6 points) Let  $\int x^3 \sqrt{1-x^2} dx = a(1-x^2)^{\frac{5}{2}} + b(1-x^2)^{\frac{3}{2}} + C$ . Which of the following are true?

(A)  $a = -\frac{1}{5}$  (B)  $b = -\frac{1}{3}$  (C)  $a = \frac{1}{5}$  (D)  $b = \frac{1}{3}$ .

Ans: B, C (from online test system)

**Problem 3.** (6 points) Let  $f(x) = \int_0^x x \cos(t^2) dt$ . Which of the following are true?

(A)  $f'(0) = 0$  (B)  $f'(0) = 1$  (C)  $f''(0) = 1$  (D)  $f''(0) = 2$ .

Ans: A, D

**Problem 4.** (6 points) Assume  $f(x) = e^{-2x} \left( ax^2 + bx - \frac{1}{4} \right)$  and  $f'(x) = e^{-2x} x^2$ . Which of the following are true?

- (A)  $a = -\frac{1}{2}$  (B)  $a = \frac{1}{2}$  (C)  $b = -\frac{1}{2}$  (D)  $b = 0$ .

Ans: A, C (from online test system)

**Problem 5.** (6 points) Let  $a$  be the area of the region between the curves  $y = x^4$  and  $y = 2x - x^2$ . Which of the following intervals contain  $a$ ?

- (A)  $[0, 0.3]$  (B)  $[0.3, 0.5]$  (C)  $[0.4, 0.7]$  (D)  $[0.2, 0.4]$

Ans: B, C

**Problem 6.** (6 points) Assume  $a = \int_0^1 \frac{1}{\sqrt{x}} dx$ . Which of the following intervals contain  $a$ ?

- (A)  $[1, 3]$  (B)  $[1.5, 3.5]$  (C)  $[2.5, 4.5]$  (D)  $[3, 5]$

Ans: A, B

## Part II. Single answer questions. ( 單選題 )

Select only ONE correct choice from a list of four choices.

**Problem 1.** (5 points) Assume  $g'(x) = e^x + x^e - e$  and  $g(0) = 0$ . Find  $g(1)$ .

- (A)  $\frac{-1}{e}$  (B)  $e - 1$  (C)  $\frac{-e}{e+1}$  (D)  $\frac{2e}{e+1}$ .

Ans: C

**Problem 2.** (5 points) Assume  $g(x) = \int \frac{\cos x}{1+\sin x} dx$  and  $g(0) = 0$ . Find  $g\left(\frac{\pi}{2}\right)$ .

(A) 0 (B)  $\sqrt{2}$  (C)  $\ln 2$  (D) 2.

Ans: C

**Problem 3.** (5 points) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{3\sin x \cos x}{\sqrt{1+3\sin^2 x}} dx$ .

(A) 0 (B) 1 (C) 2 (D) 3

Ans: B(from online test system)

**Problem 4.** (5 points) Evaluate  $\int_0^1 (x+2)(x-3) dx$ .

(A)  $-\frac{25}{4}$  (B)  $-\frac{31}{6}$  (C)  $-\frac{37}{6}$  (D)  $\frac{25}{4}$

Ans: C

**Problem 5.** (5 points) Evaluate  $\int_{-1}^1 \left(2 + \frac{\tan x}{1+x^2+x^4}\right) dx$

(A) 0 (B) 2 (C) 3 (D) 4

Ans: D

**Problem 6.** (5 points) Find  $\int_1^e \frac{\ln x}{x^2} dx$ .

(A)  $1 - \frac{1}{e}$  (B)  $1 - \frac{2}{e}$  (C)  $2 - \frac{1}{e}$  (D)  $2 - \frac{2}{e}$

Ans: B

**Problem 7.** (5 points) Find  $\int_1^9 \frac{2^{\sqrt{x}}}{\sqrt{x}} dx$ .

- (A)  $\frac{6}{\ln 2}$  (B)  $\frac{12}{\ln 2}$  (C)  $\frac{24}{\ln 2}$  (D)  $\frac{48}{\ln 2}$ .

Ans: B

**Problem 8.** (6 points) Find the equation of tangent line of the graph of  $y^x = x^y$  at the point

(1,1).

(A)  $y - 1 = -1(x - 1)$  (B)  $y - 1 = -2(x - 1)$

(C)  $y - 1 = 1(x - 1)$  (D)  $y - 1 = 2(x - 1)$ .

Ans: C(from online test system)

**Problem 9.** (6 points) Find  $\int e^{2x} \cos x \, dx$

(A)  $\frac{1}{5}e^{2x} \sin x + \frac{1}{5}e^{2x} \cos x + C$  (B)  $e^{2x} \sin x + e^{2x} \cos x + C$

(C)  $\frac{1}{5}e^{2x} \sin x + \frac{2}{5}e^{2x} \cos x + C$  (D)  $\frac{7}{5}e^{2x} \sin x + \frac{3}{5}e^{2x} \cos x + C$ .

Ans: C(from online test system)

**Problem 10.** (5 points) Find  $\int_0^{\infty} e^{-x} dx$ .

(A) -2 (B) -1 (C) 0 (D) 1

Ans: D

**Problem 11.** (6 points) Find the area of the region R under  $y = x^4 - 2x^3 + 2$  between  $x = -1$

and  $x = 2$ .

(A) 4 (B) 4.8 (C) 5.1 (D) 6.4

Ans: C

**Problem 12.** (6 points) Find the volume of the solid of revolution obtained by revolving the plane

region R bounded by  $y = \sqrt{x}$ , the  $x$ -axis and the line  $x = 4$  above the  $x$ -axis.

(A)  $6\pi$  (B)  $8\pi$  (C)  $10\pi$  (D)  $7\pi$

Ans: B