On the generalized Alon-Frankl- Loász theorem

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Let $[n]$ denote the set $\{1, 2, \ldots, n\}$. The *Kneser graph*, denoted as $KG(n, k)$, is defined for positive integers $n \geq 2k$ as the graph having the collection of all k-subsets of $[n]$ as vertex set. Two vertices are defined to be adjacent in $KG(n, k)$ if they are disjoint. Kneser (1955) conjectured that for positive integers n and k with $n \geq 2k$, the chromatic number of Kneser graphs is equal to $n-2(k-1)$. The Kneser conjecture was proved by Lovász (1978) using the Borsuk-Ulam theorem; all subsequent proofs, extensions and generalizations also relied on Algebraic Topology results, namely the Borsuk-Ulam theorem and its extensions.

For any positive integer $r \geq 2$, the *Kneser hypergraph* $KG^r(n, k)$ is an r-uniform hypergraph which has the collection of all k-subsets of $[n]$ as vertex set and whose edges are formed by the r-tuples of disjoint k-element subsets of $[n]$. Choosing $r = 2$, we obtain the ordinary Kneser graph $KG(n, k)$. Erdős (1976) conjectured that, for $n \geq rk$,

$$\chi\left(KG^r(n, k)\right) = \left\lceil \frac{n - r(k-1)}{r-1} \right\rceil.$$

The Erdős-conjecture settled by Alon, Frankl and Lovász (1986).

In this talk, we investigate the general results about the Alon-Frankl-Lovász theorem.