Rainbow Graph Designs

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A k-edge-coloring of a graph G is a mapping from $E(G)$ into $\{1, 2, \ldots, k\}$. If, in addition, incident edges of G receive distinct colors, then the coloring is a proper edge-coloring. A subgraph H of an edge-colored graph G is a rainbow subgraph provided all the edges of H are of distinct colors.

An H-design of G is a decomposition of G such that all its members are isomorphic to H, denoted by $H|G$. Furthermore, if G is edge-colored and each member H is a rainbow subgraph, then we have a rainbow H-design of G, denote by $H|_R G$. In case that $G \cong K_n$, we simply call it a rainbow H-design of order n.

We are interested in the following problems.

- **Problem 1.** Can we find a $\chi'(G)$-edge-coloring and an H-decomposition of G such that each member of the decomposition is a rainbow subgraph?
- **Problem 2.** Given a proper edge-coloring of G, can G be decomposed into subgraphs such that each member is isomorphic to H and also each member is a rainbow subgraph?

Both of the above problems are easy to solve if $G \cong K_n$ and the subgraphs do have certain structure, for example triangles and stars. But, it won't be that trivial if we have a larger subgraph to consider. Since our focus is on complete graph of order n, the H-decomposition obtained in Problem 1 will be referred to as a "weak" rainbow H-design of order n and the one obtained in Problem 2 is a "strong" rainbow H-design of order n. In this talk, I shall report some progress of working on these two problems.