Folding Phenomenon of Major-balance Identities on Restricted Involutions

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Simion and Schmidt initiated the study of sign-balance for pattern-avoiding permutations, who proved that the number of even and odd 321-avoiding permutations of length n are equal if n is even, and differ (up to a sign) by a Catalan number otherwise. Adin and Roichman obtained a refined sign-balance result, respecting the position of the last descent of a permutation. The refined result reveals an interesting folding phenomenon (up to small variations) that the signed enumerator of objects of size $2n$ is essentially equal to the ordinary enumerator of objects of size n. In this talk, we present refined major-balance identities on the 321-avoiding involutions of length n, respecting the leading element of permutations. The proof is based on sign-reversing involutions on the lattice paths within a $\lfloor \frac{n}{2} \rfloor \times \lceil \frac{n}{2} \rceil$ rectangle. Moreover, we prove affirmatively a question about refined major-balance identities on the 123-avoiding involutions, respecting the number of descents. This talk is based on joint work with Hsiang-Chun Hsu and Hsin-Chieh Liao.