Signed Mahonian Identities on Permutations with Subsequence Restrictions

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In this talk, we present a number of results surrounding Caselli's conjecture on the equidistribution of the major index with sign over the two subsets of permutations of $\{1, 2, \ldots, n\}$ containing respectively the word $1\,2\cdots k$ and the word $(n-k+1)\cdots n$ as a subsequence, under a parity condition of n and k. We derive broader bijective results on permutations containing varied subsequences. As a consequence, we obtain the signed mahonian identities on families of restricted permutations, in the spirit of a well-known formula of Gessel–Simion, covering a combinatorial proof of Caselli's conjecture. We also derive an extension of the insertion lemma of Han and Haglund–Loehr–Remmel which allows us to obtain a signed enumerator of the major-index increments resulting from the insertion of a pair of consecutive numbers in any place of a given permutation.