Approximation algorithms for solving the 1-line Euclidean minimum Steiner tree problem

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In this talk, we consider the 1-line Euclidean minimum Steiner tree problem, which is a variation of the Euclidean minimum Steiner tree problem and defined as follows. Given a set $P = \{r_1, r_2, \ldots, r_n\}$ of n points in the Euclidean plane $\mathbb{R}^2$, we are asked to find the location of a line l and an Euclidean Steiner tree $T(l)$ in $\mathbb{R}^2$ such that at least one Steiner point is located at such a line l, the objective is to minimize total weight of such an Euclidean Steiner tree $T(l)$, *i.e.*, $\min\{\sum_{e \in T(l)} w(e) | T(l)$ is an Euclidean Steiner tree as mentioned-above$\}$, where we define weight $w(e) = 0$ if the end-points u, v of each edge $e = uv \in T(l)$ are both located at such a line l and otherwise we denote weight $w(e)$ to be the Euclidean distance between u and v. Given a fixed line l as an input in $\mathbb{R}^2$, we refer this problem as the 1-line-fixed Euclidean minimum Steiner tree problem; In addition, when Steiner points added are all located at such a fixed line l, we refer this problem as the constrained Euclidean minimum Steiner tree problem.

We obtain the following two main results. (1) Using a polynomial-time exact algorithm to find a constrained Euclidean minimum Steiner tree, we can design a 1.214-approximation algorithm to solve the 1-line-fixed Euclidean minimum Steiner tree problem, and this algorithm runs in time $O(n \log n)$; (2) Using the algorithm designed in (1) for many times, a technique of finding linear facility location and an important lemma proved by some techniques of computational geometry, we can provide a 1.214-approximation algorithm to solve the 1-line Euclidean minimum Steiner tree problem, and this new algorithm runs in time $O(n^3 \log n)$.

Keywords: Euclidean minimum Steiner tree, Constrained Euclidean minimum Steiner tree, Steiner ratio, Approximation algorithms and Complexity.

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