A new construction of equiangular lines from integral lattices

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Let X be an equiangular set of r lines in the Euclidean space $\mathbb{R}^n$ with angle $1/5$, where $r > n$. Since the Seidel matrix S of X has minimum eigenvalue -5, there is a subset of norm-3 vectors from some integral lattice whose Gram matrix can be constructed from S. By a theorem of Conway and Sloane [1], we only need to look for unimodular odd lattices of ranks at most $n+3$.

As a concrete example, we look for equiangular line sets in $\mathbb{R}^{14}$. The dual lattice of E_7^2 contains 3,136 vectors of norm 3. We make a graph G whose vertices are these norm-3 vectors, and two vertices are adjacent if and only if their inner product is either 0 or 1. Inside G we find a 28-clique, which produces 28 equiangular lines in $\mathbb{R}^{14}$. Interestingly, this equiangular line set is not isomorphic with Tremain's example [2] that comes from a $(7,3,1)$-design.

This is joint work with Gary Greaves, Jack H. Koolen, and Wei-Hsuan Yu.

References

[1] John H. Conway and N. J. A. Sloane, *Low-dimensional lattices V. Integral coordinates for integral lattices*, Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences, **426**(1871): 211–232, 1989.

[2] Janet C. Tremain, *Concrete constructions of real equiangular line sets*, arXiv preprint, `arXiv:0811.2779`, 1–39, 2008.