

# Clustered coloring for old coloring conjectures

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Hadwiger (Hajós and Gerards and Seymour, respectively) conjectured that the vertices of every graph with no  $K_{t+1}$  minor (topological minor and odd minor, respectively) can be colored with  $t$  colors such that any pair of adjacent vertices receive different colors. These conjectures are stronger than the Four Color Theorem and are either wide open or false in general. A weakening of these conjectures is to consider clustered coloring which only requires every monochromatic component to have bounded size instead of size 1. It is known that  $t$  colors are still necessary for the clustered coloring version of those three conjectures. Joint with David Wood, we prove a series of tight results about clustered coloring on graphs with no subgraph isomorphic to a fixed complete bipartite graph. These results have a number of applications. In particular, they imply that the clustered coloring version of Hajós' conjecture is true for bounded treewidth graphs in a stronger sense:  $K_{t+1}$  topological minor free graphs of bounded treewidth are clustered  $t$ -list-colorable. They also lead to the first linear upper bound for the clustered coloring version of Hajós' conjecture and the currently best upper bound for the clustered coloring version of the Gerards-Seymour conjecture.