

# Problems and results on permutations

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In this talk we will introduce some new problems and results on permutations. If  $p = 2n + 1$  is an odd prime, then the list  $1^2, \dots, n^2$  is a permutation of all the  $n$  quadratic residues  $a_1 < \dots < a_n$  among  $1, \dots, p - 1$ , and we determine its sign in the case  $p \equiv 3 \pmod{4}$ . For any positive integer  $n$ , we show that there is a unique permutation  $\pi$  of  $\{1, \dots, n\}$  such that all the numbers  $k + \pi(k)$  ( $k = 1, \dots, n$ ) are powers of two. The speaker conjectured that if a group  $G$  contains no element of order among  $2, \dots, n + 1$  then any  $A \subseteq G$  with  $|A| = n$  can be written as  $\{a_1, \dots, a_n\}$  with  $a_1, a_2^2, \dots, a_n^n$  pairwise distinct; when  $G$  is a torsion-free abelian group we confirm this via Alon's Combinatorial Nullstellensatz.