Antimagic Labeling Problems on Graphs

National Taiwan Normal University
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2019/08/20
2019年圖論與組合數學國際研討會 暨 第十屆海峽兩岸圖論與組合數學研討會

2005.06.23 台灣新竹 黃光明老師與我的家人

2017.08.10 匈牙利 陳宏賓(台灣中興大學)、李渭天(台灣中興大學)

2013.09.台灣花蓮 黃瑜培(北京師範大學珠海校區)、郭君逸(台灣師範大學)

2018.08.24 台灣花蓮 六十石山金針花季 潘志實 與其夫人 (台灣淡江大學)

2019.07 黃光明老師與我 合影於 舊金山

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Vertex Sum accompany with an edge labeling

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Edge Labeling on this talk

- $f: E(G) \to N$

Vertex Sum accompany with an edge labeling

- The vertex sum at $u \in V(G)$ accompany with f is the sum of the labels assigned to edges incident to u.

Antimagic Labeling

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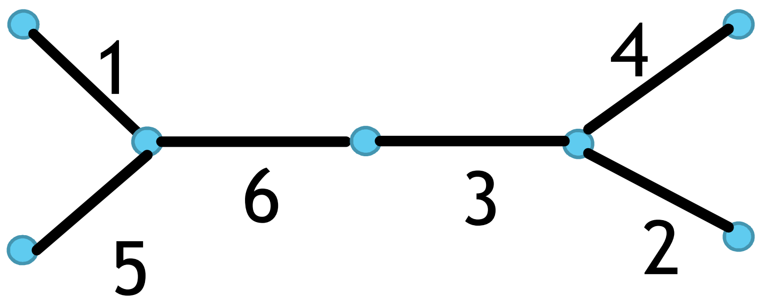
Antimagic Graph

- If G has an antimagic labeling, then G is called antimagic.

Non-antimagic Labeling

$f: E(G) \rightarrow \{1,2,\cdots,6\}$ is not an antimagic labeling.



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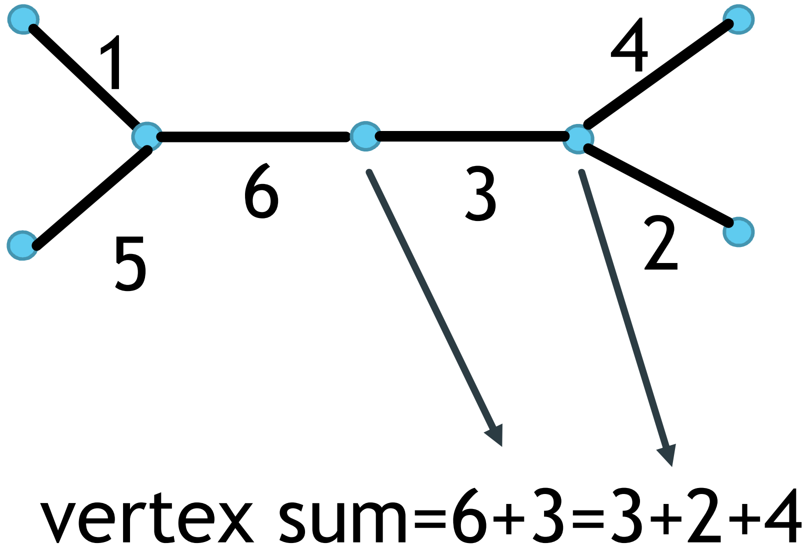
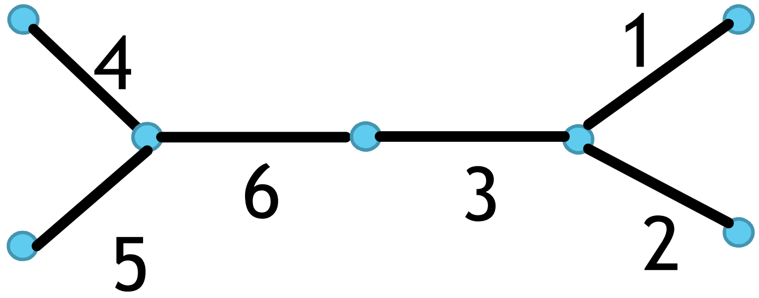


An Antimagic Graph



History

- This problem was introduced by Hartsfield and Ringel in 1990.

N. Hartsfield and G. Ringel. Pearls in Graph Theory, Academic Press, INC., Boston,1990 (revised version, 1994), 108-109.

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They put two conjectures concerning antimagic labeling of graphs.

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History

- Conjecture 1: Every connected graph other than K_2 is antimagic.

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- Conjecture 1: Every connected graph other than K_2 is antimagic.

- Conjecture 2: Every tree other than K_2 is antimagic.

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History

- Conjecture 1: Every connected graph other than K_2 is antimagic.

- Conjecture 2: Every tree other than K_2 is antimagic.

The two conjectures are still open now.

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Well Known Results

- For conjecture 1:

Complete graphs, cycles, wheels and complete bipartite graphs are antimagic.

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- Alon, Kaplan, Lev, Roditty and Yuster [2004]

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Dense graphs are antimagic.
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Graphs with minimum degree $\delta(G) > \Omega(\log |V(G)|)$ or maximum degree $\Delta(G) > |V(G)| - 2$ are antimagic.

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Well Known Results

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Regular graphs are antimagic. [2015,2016]

K. Berczi, A. Bernath, and M. Vizer. Regular graphs are antimagic. The Electronic Journal of Combinatorics 22 (2015)

F. Chang, Y.-Ch. Liang, Z. Pan, and X. Zhu. Antimagic labeling of regular graphs. J. Graph Theory 82 (2016), 339-349.

Well Known Results

- For conjecture 2:

  Paths and stars are antimagic.

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  Every tree with at most one vertex of degree 2 is antimagic.

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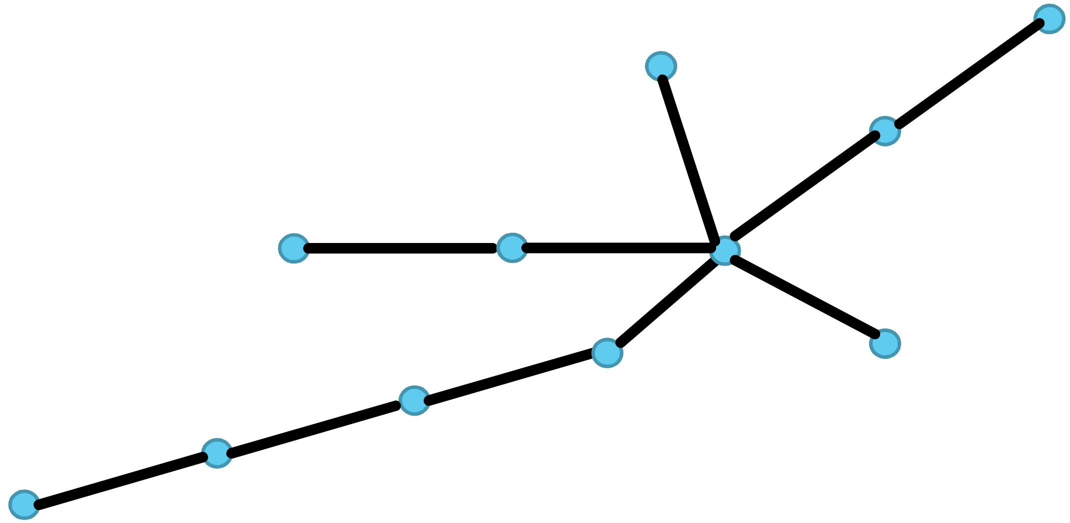
- Liang, Wong and Zhu [2014] corrected this error.

G. Kaplan, A. Lev, and Y. Roditty. On zero-sum partitions and antimagic trees. Discrete Math., 309, (2009), 2010-2014.
Y.-Ch. Liang, T.-L. Wong and X. Zhu. Antimagic labeling of trees. Discrete Math.,331, (2014), 9-14.

Well Known Results

- Shang [2015] proved spiders are antimagic.

A spider is a tree with one vertex of degree at least 3.



J.-L. Shang, Spiders are antimagic, Ars Combinatoria, 118 (2015), 367-372.

Strongly Antimagic Graph

T.-M. Wang and C. C. Hsiao, On anti-magic labeling for graph products, Discrete Math. 308(16), (2008), 3624-3633.

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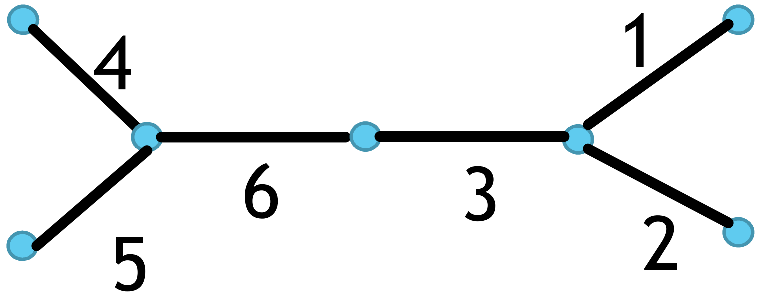

An antimagic labeling, but Non-strongly antimagic labeling

k-shifted Antimagic Labeling

If $f: E(G) \rightarrow \{k+1, \cdots, m+k\}$ is an injective function

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If $f: E(G) \to \{k+1, \cdots, m+k\}$ is an injective function such that all vertex sums are pairwise distinct,

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If $f: E(G) \to \{k+1, \cdots, m+k\}$ is an injective function such that all vertex sums are pairwise distinct, then f is called an k-shifted antimagic labeling on G. And G is called k-shifted antimagic.

Theorem: If G is strongly antimagic, then $\forall k \in N$, G is k-shifted antimagic.

k-shifted antimagic$\Longrightarrow$? $(k+1)$−shifted antimagic ?

Question 1 ?

k-shifted antimagic$\Longrightarrow ? \, (k+1)$−shifted antimagic ?

Question 1 ?

Is there a k-shifted antimagic graph but not $(k+1)$-shifted antimagic?

- Kaplan, Lev and Roditty [2009]

Every tree except K_2 with at most one vertex of degree 2 is antimagic.

Question 2 ?

- Kaplan, Lev and Roditty [2009]

Every tree except K_2 with at most one vertex of degree 2 is antimagic.

Question 2 ?

Is a tree with at most one vertex of degree 2 strongly antimagic?

Question 3 ?

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Question 2 ?

Is a tree with at most one vertex of degree 2 strongly antimagic?

Question 3 ?

Is there a connected graph except K_2 not strongly antimagic?

Disconnected graphs?

$3P_3 \cup C_3$ is not strongly antimagic

- Li and Silalahi, Master Thesis

Antimagic Labelings on Disconnected Graphs.

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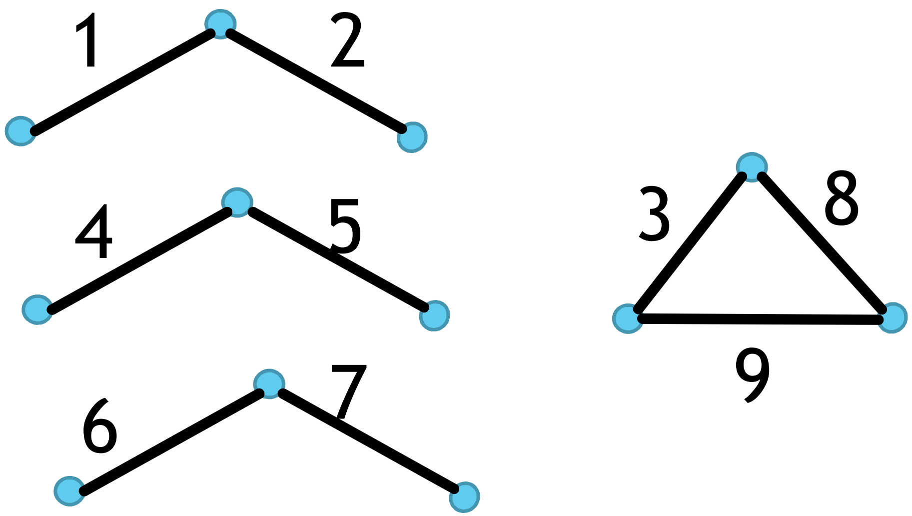

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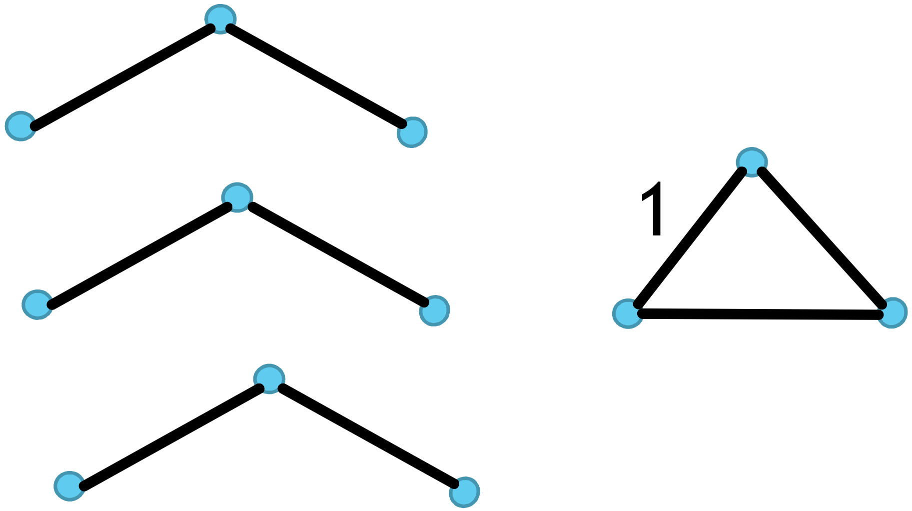

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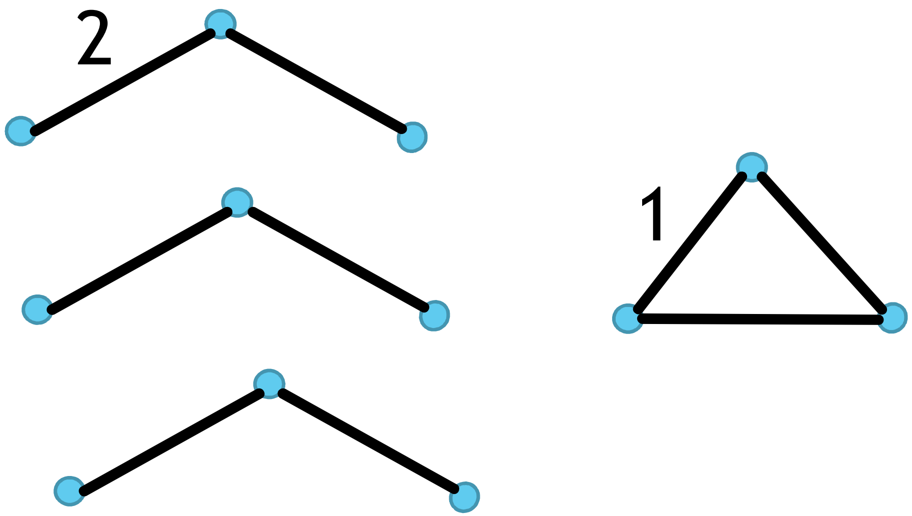

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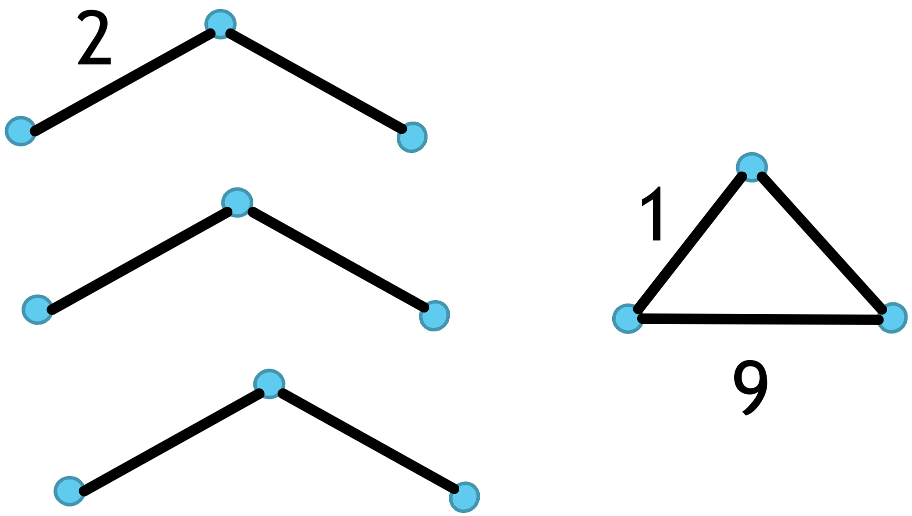

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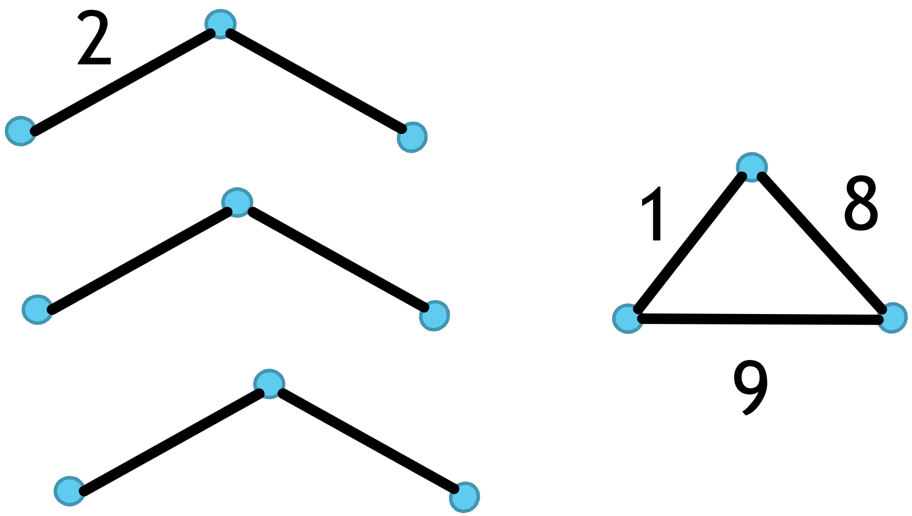

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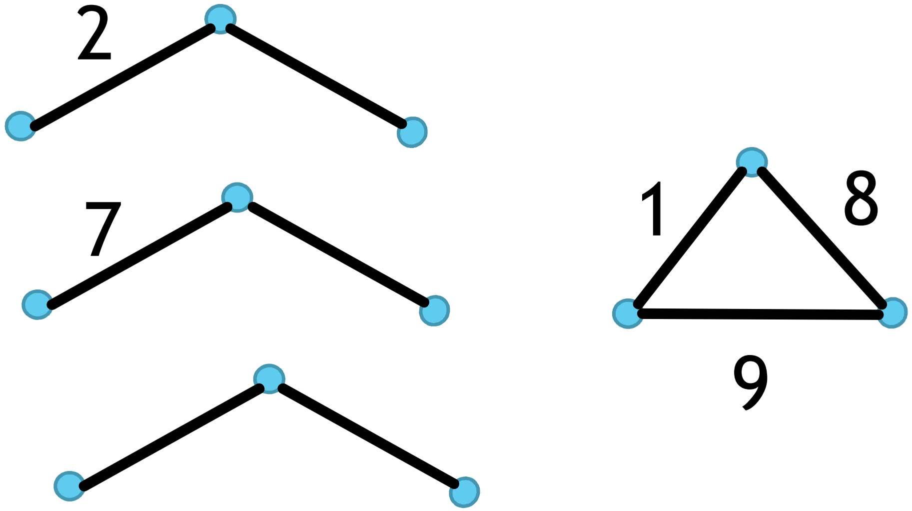

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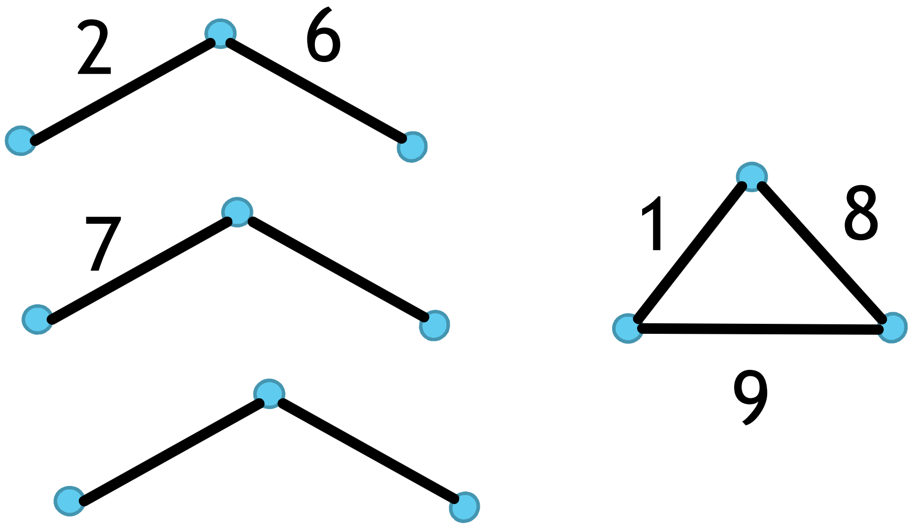

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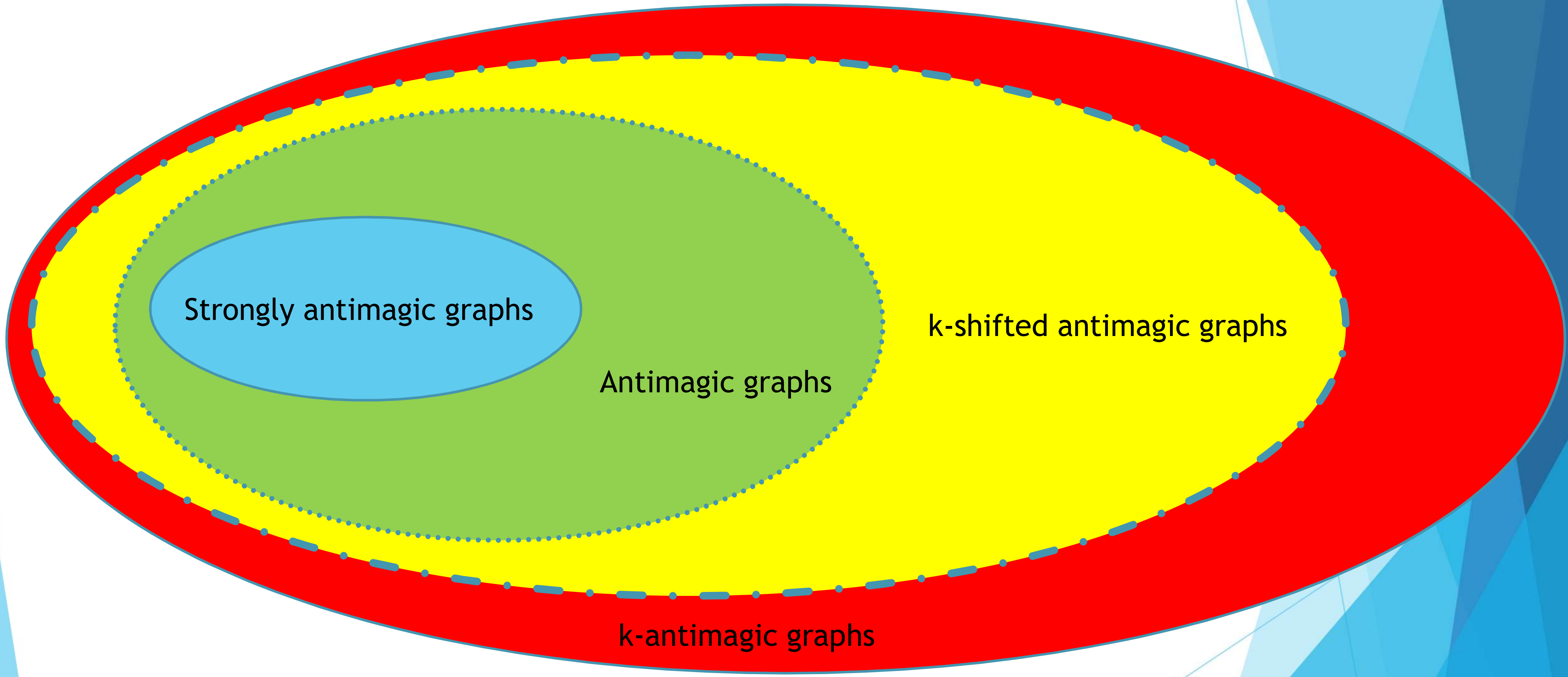
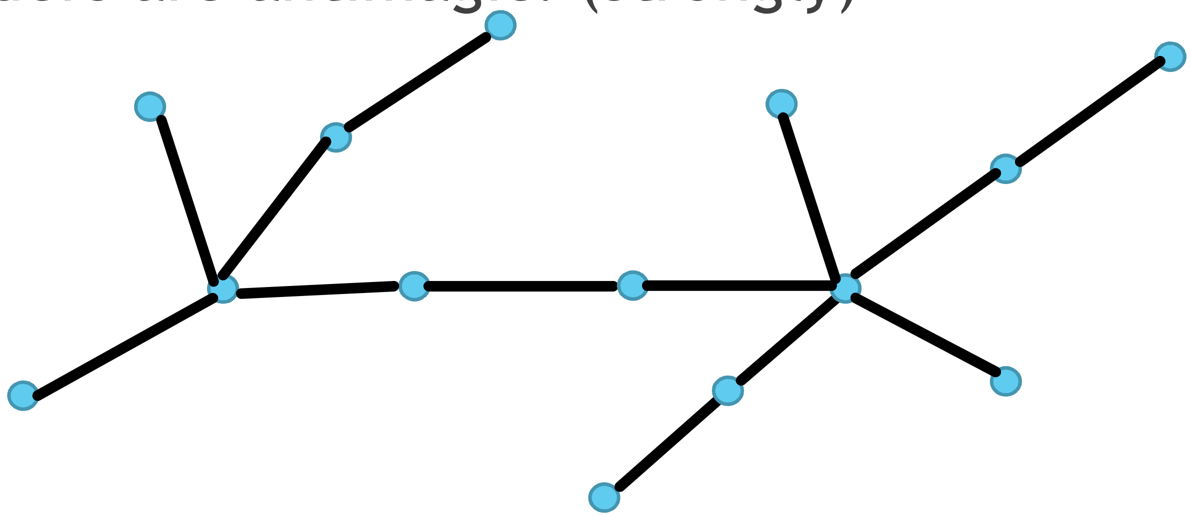


k+1-shifted antimagic graphs ?

Our Results

- Chang, Kin, Li and Pan [2018^+]

Double spiders are antimagic. (strongly)



- Guo, Li and Chang [preprint]

Complete multipartite graphs are strongly antimagic.

Question ?

Is every connected graph other than K_2 shifted-antimagic?

- Chang, Chen, Li, Pan [2018^+]

Theorem: Trees are shifted-antimagic.

Thank you for your attention!!

For a double spider, we decompose its edge set into three subsets: The core path P^{core},

Figure 1: A double spider $DS(L, P^{core}, R)$.

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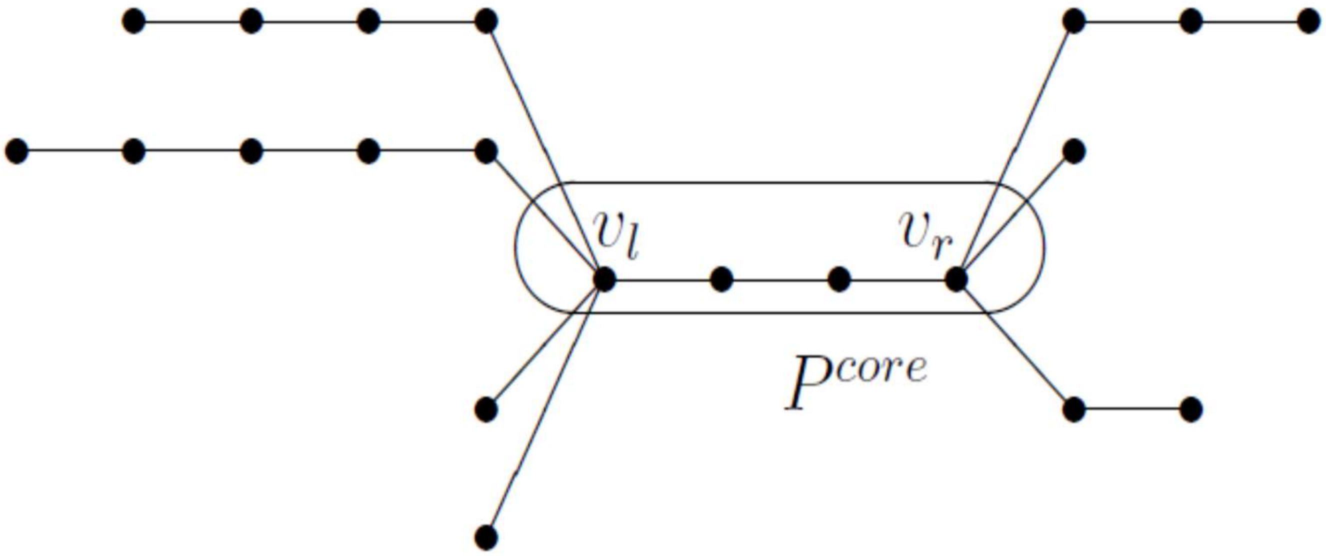
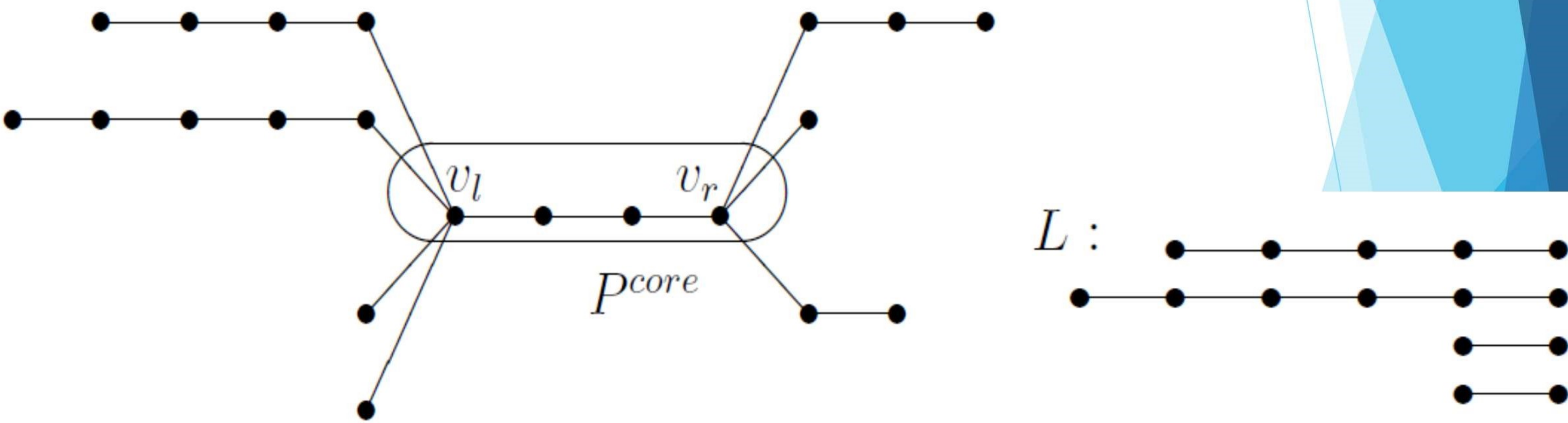


Figure 1: A double spider $DS(L, P^{core}, R)$.

For a double spider, we decompose its edge set into three subsets: The core path P^{core}, L and



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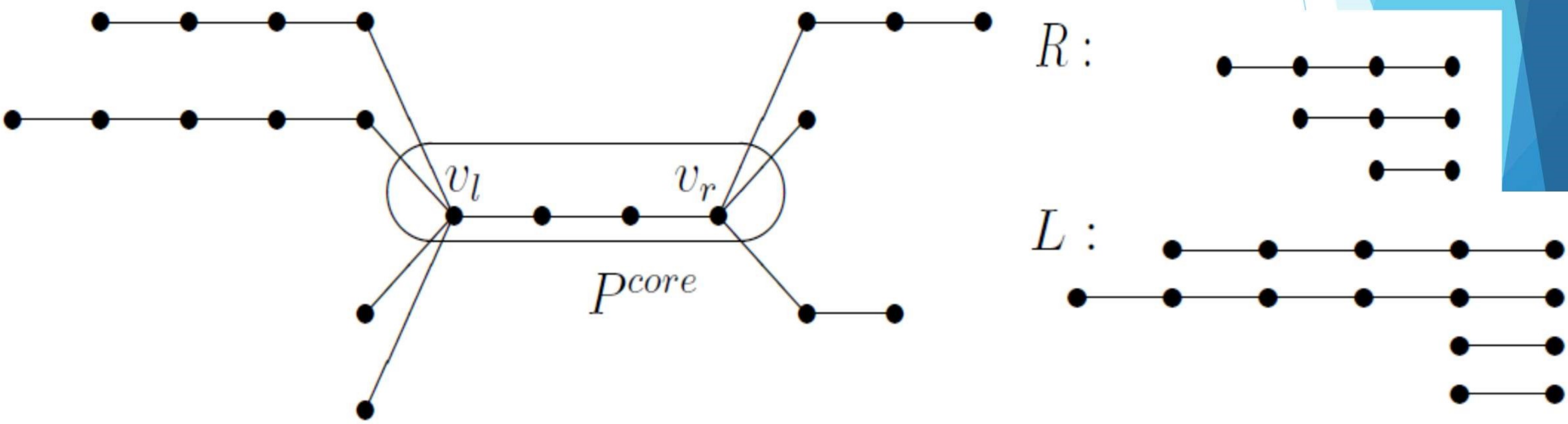
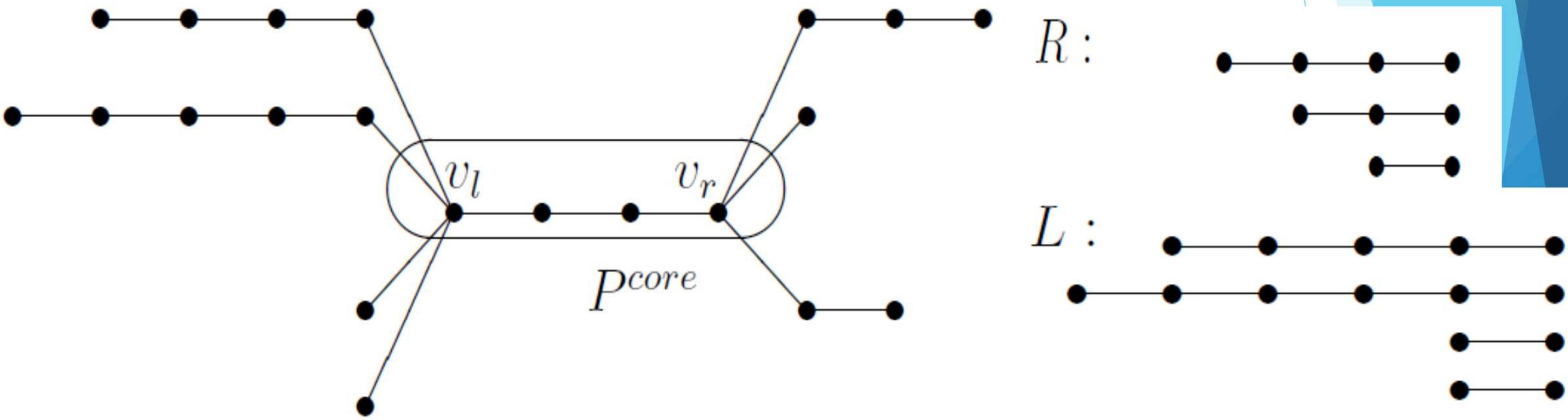


Figure 1: A double spider $DS(L, P^{core}, R)$.

For a double spider, we decompose its edge set into three subsets: The core path P^{core}, L and R.
We denote the endpoints of P^{core} by v_l and v_r, respectively and assume L contains at least as many paths as R, hence $\deg(v_l) \geq \deg(v_r)$.



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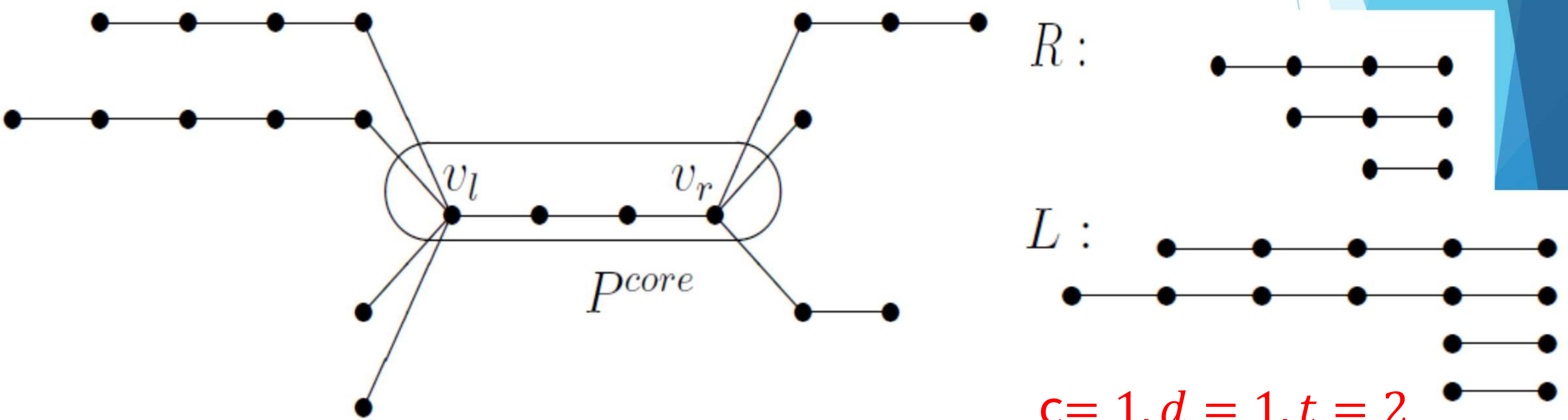
$a = 2, b = 1$



c$= 1, d = 1, t = 2$

Figure 1: A double spider $DS(L, P^{core}, R)$.

Lemma 4 *If* $\deg(v_l) = \deg(v_r) = 3$

then $DS(L, P^{core}, R)$ *is strongly antimagic.*

Lemma 5 *If* $\deg(v_l) > \deg(v_r) \geq 3$, $b = 0$,

and R *has no odd path of length at least* 3,

then $DS(L, P^{core}, R)$ *is strongly antimagic.*

Lemma 6 *If* $\deg(v_l) > \deg(v_r) \geq 3$, $b = 0$, *and* R *has at least one odd path of length at least* 3, *then* $DS(L, P^{core}, R)$ *is strongly antimagic.*

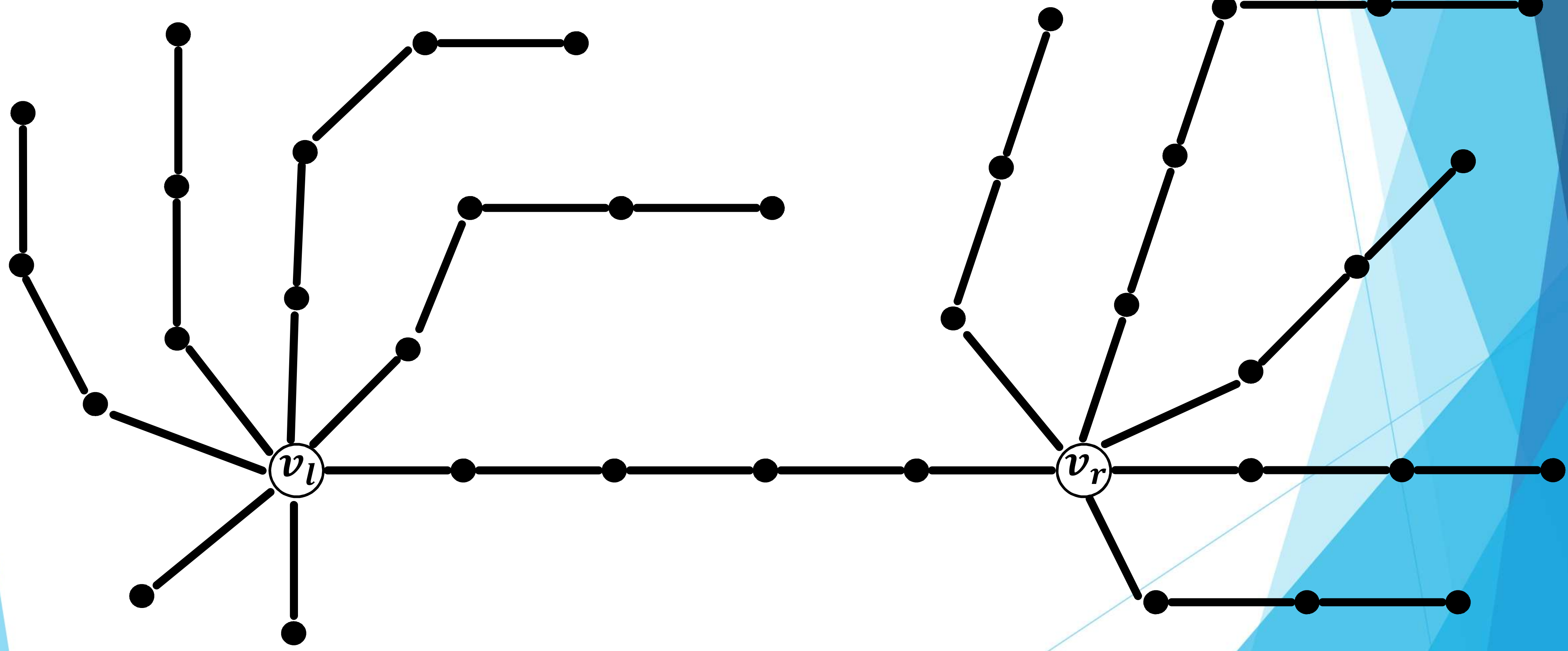
Lemma 7 *If* $\deg(v_l) > \deg(v_r) \geq 3$ *and* $b \geqslant 1$, *then* $DS(L, P^{core}, R)$ *is strongly antimagic.*

Lemma 6 *If* $\deg(v_l) > \deg(v_r) \geq 3$, $b = 0$, *and* R *has at least one odd path of length at least* 3, *then* $DS(L, P^{core}, R)$ *is strongly antimagic.*

Proof of Lemma 6:

We construct a bijective mapping f by assigning $1, 2, \ldots, m$ to the edges accordingly in the following steps.

Step 1. Label the odd edges of the odd paths in R



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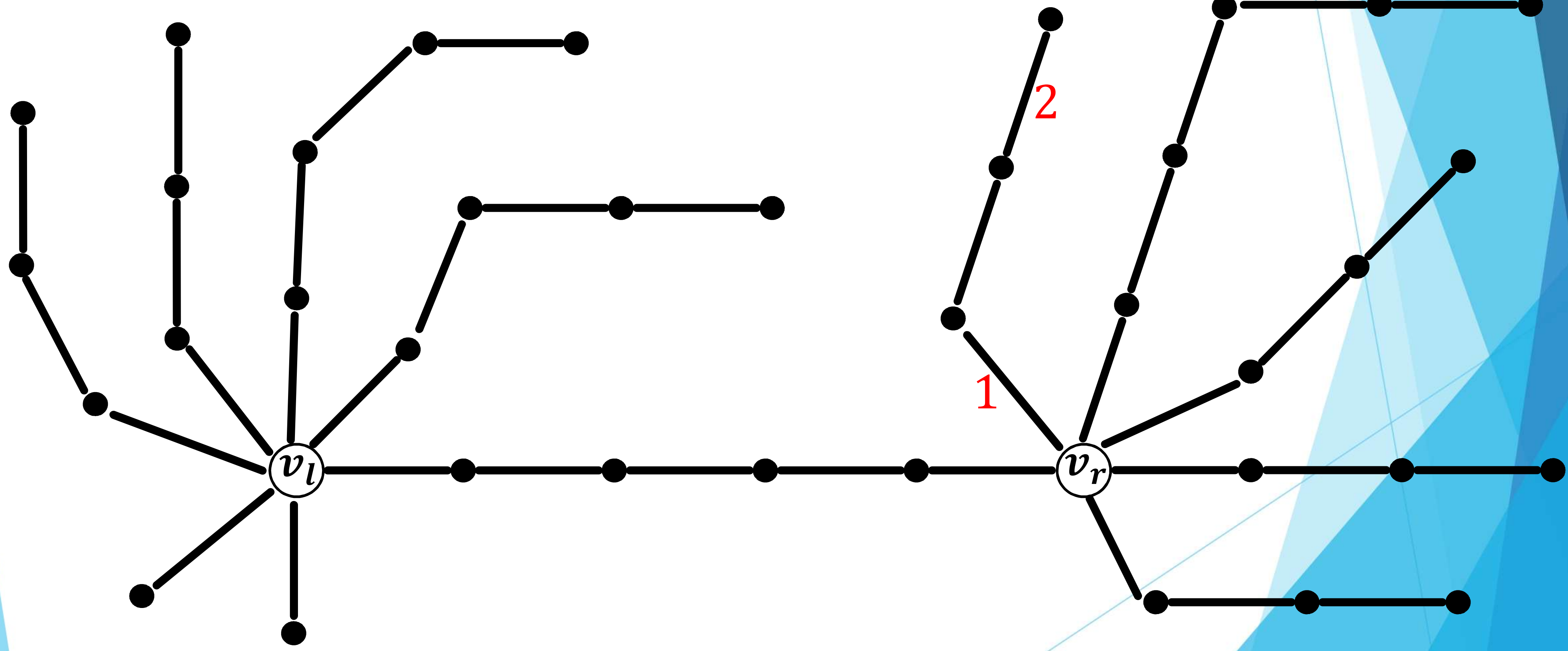

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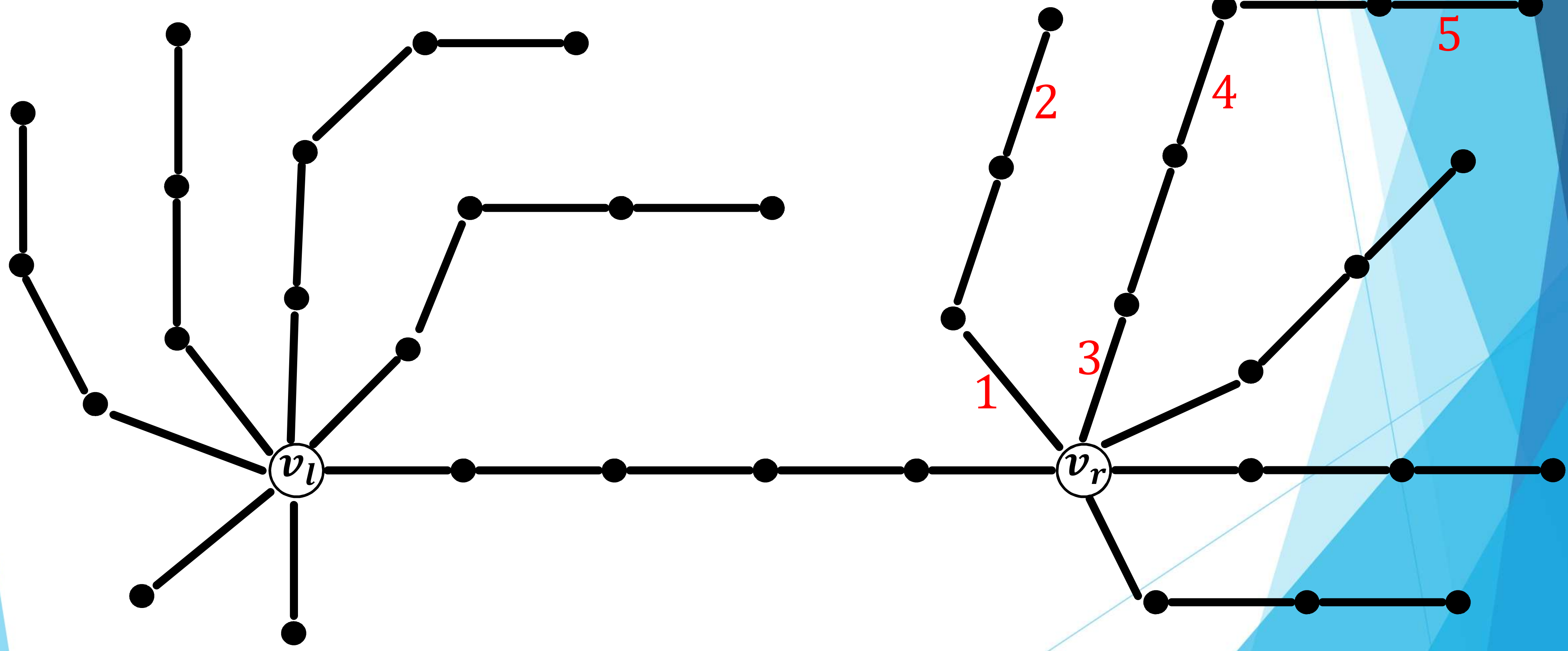

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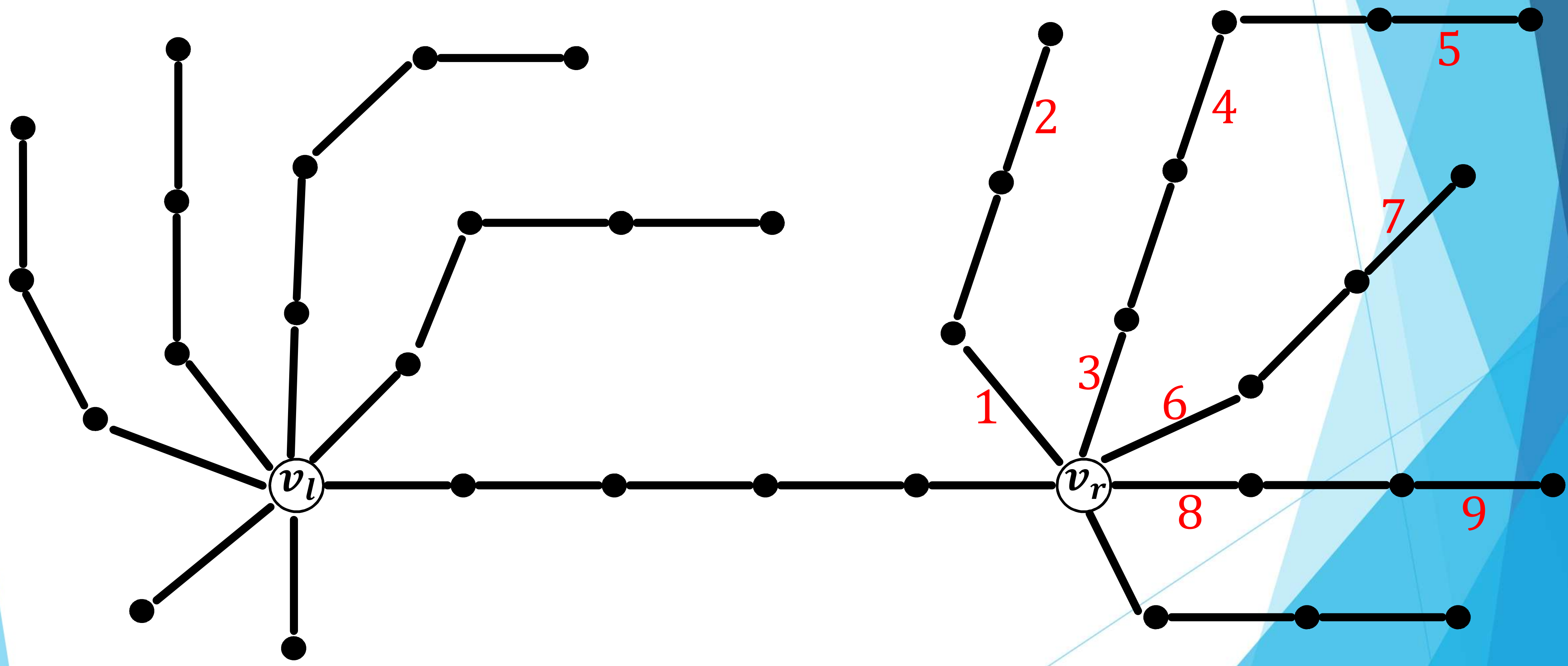
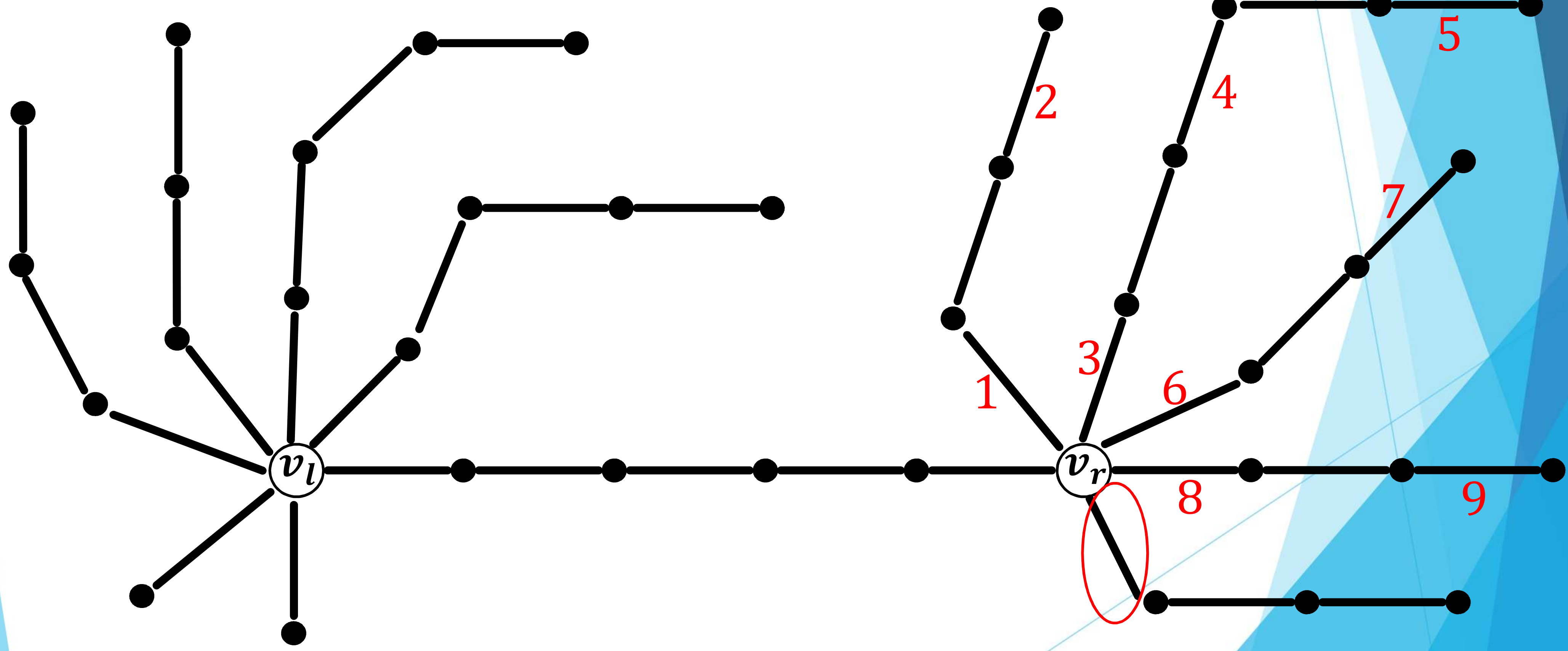


Step 1. Label the odd edges of the odd paths in R

We will label the edge $e_{a,1}^{r,odd}$ later in order to ensure that the vertex sum at v_r is large enough.



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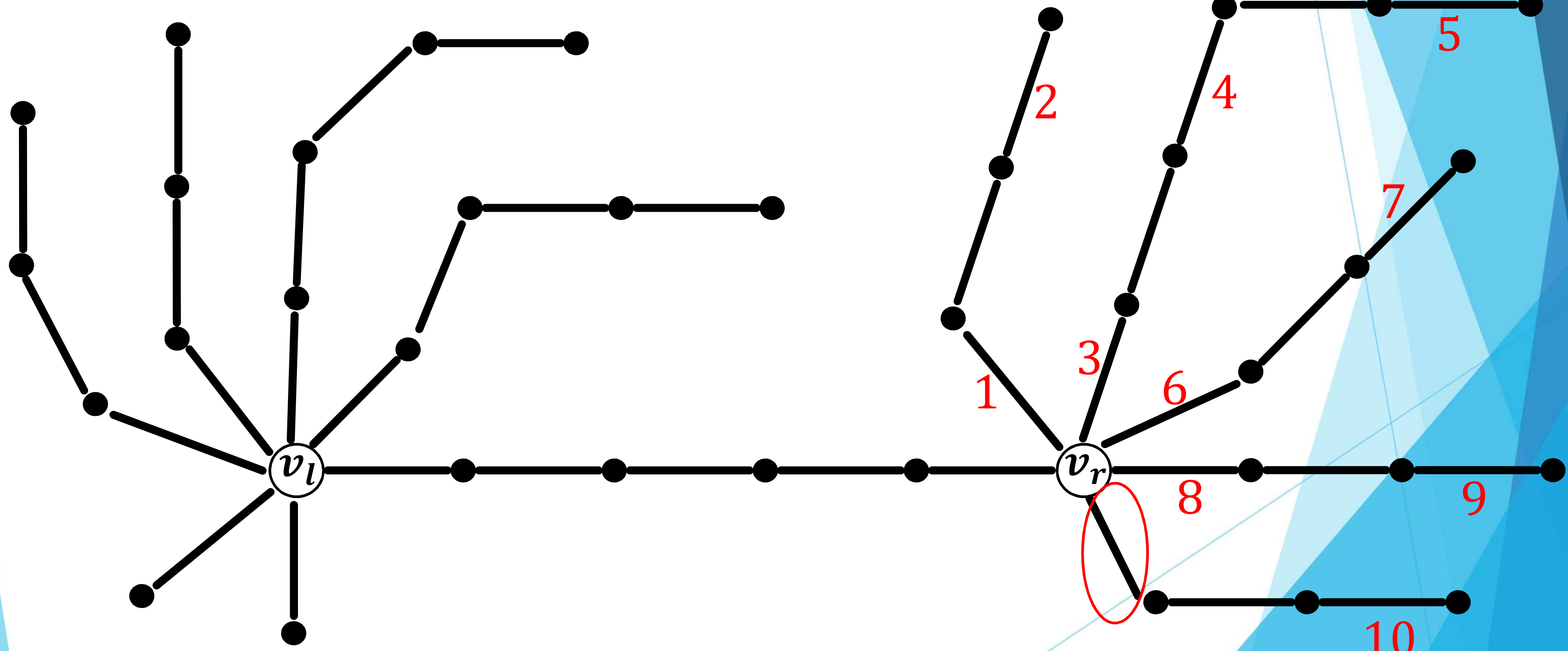
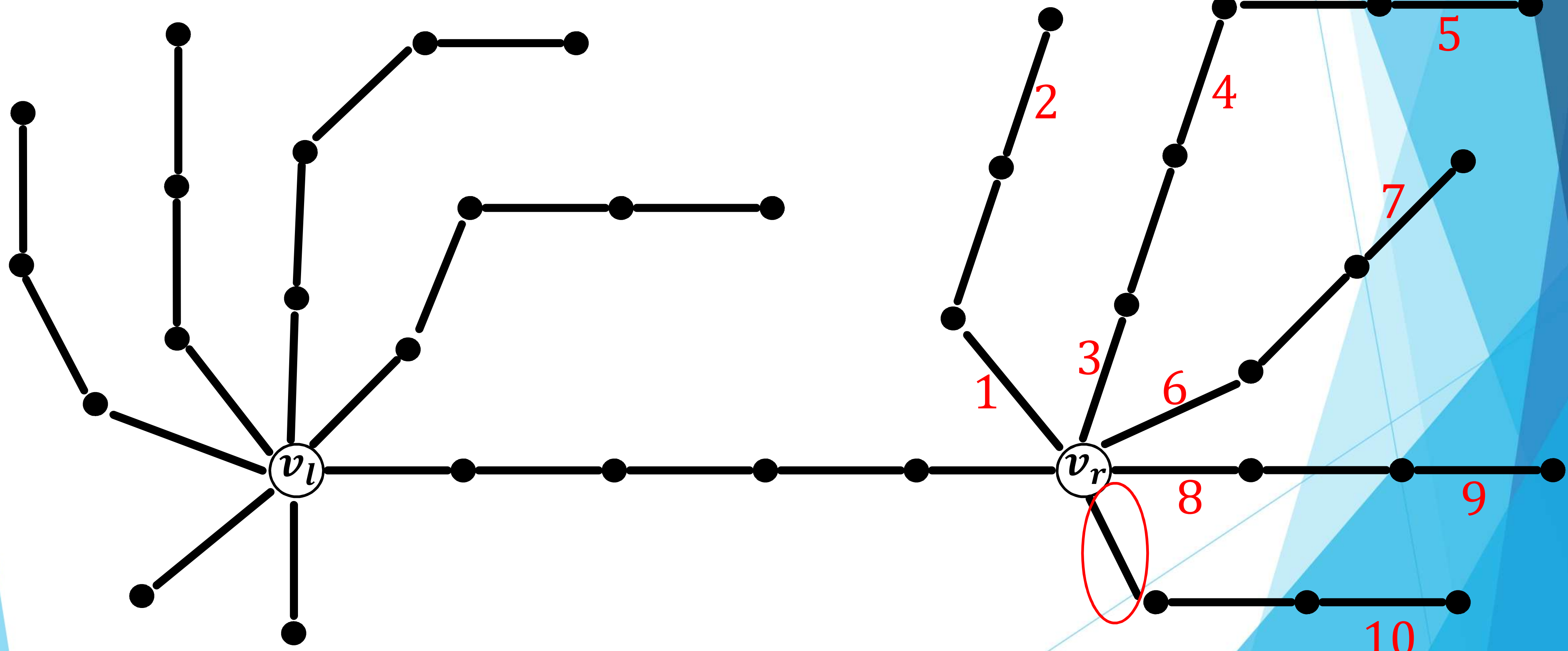


Step 2.

Label the odd edges of the odd paths with length at least 3 in L.

We also leave the c edges $e^{l,odd}_{i,2w_i+1}$ for $1 \leq i \leq c$ to enlarge the vertex sum at v_l.



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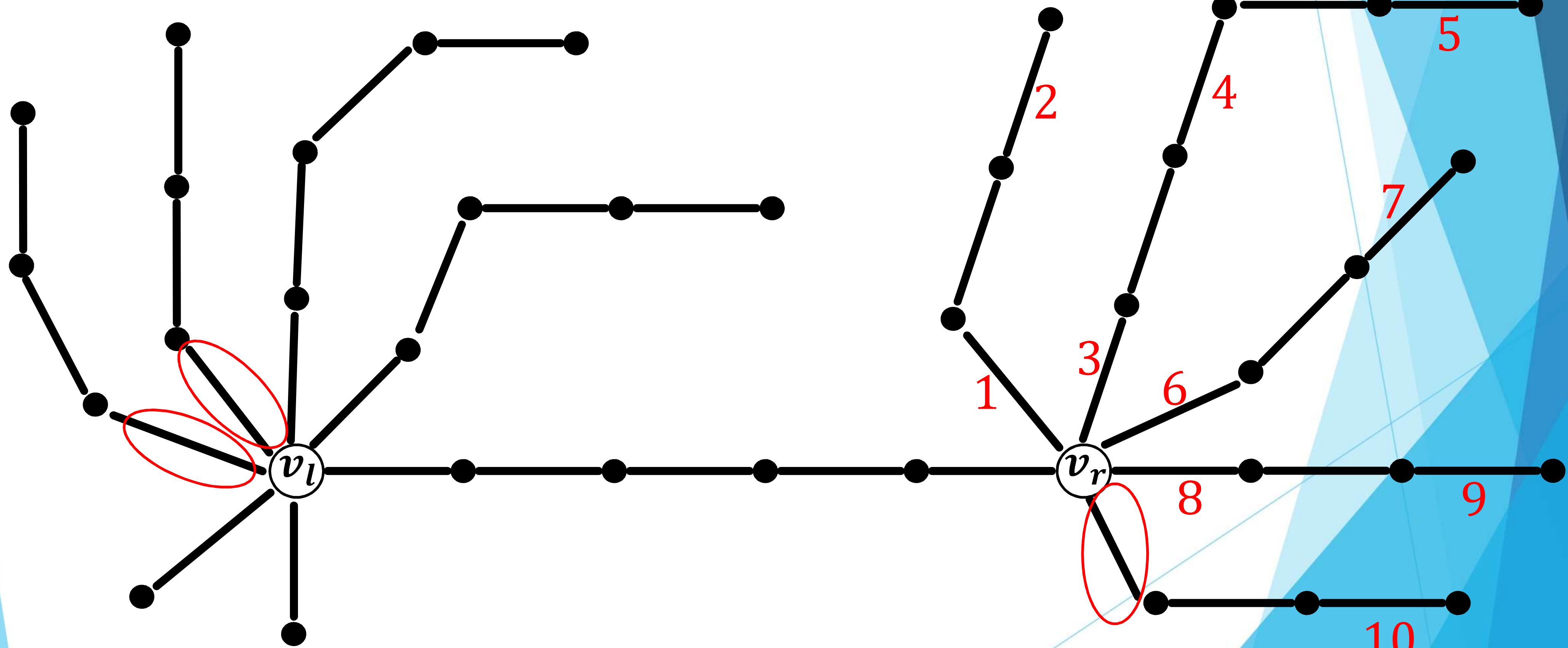

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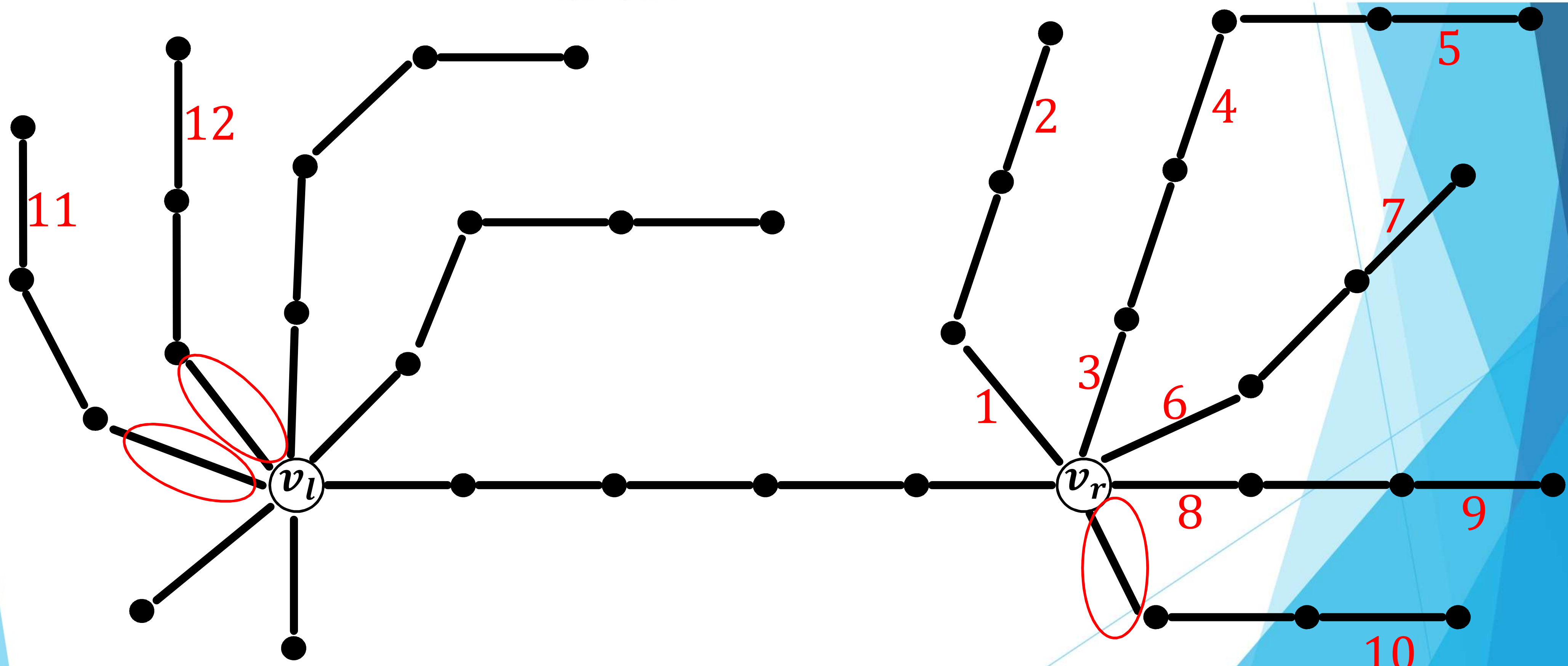
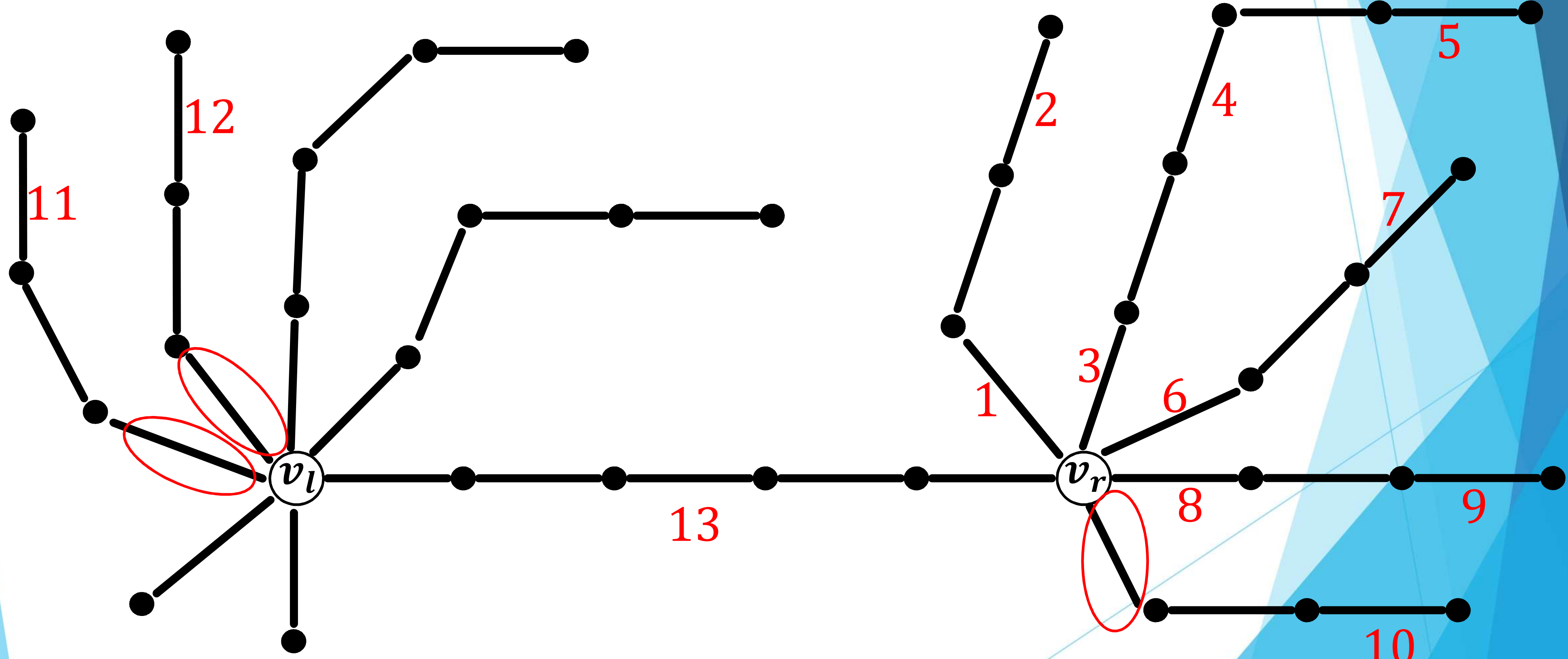
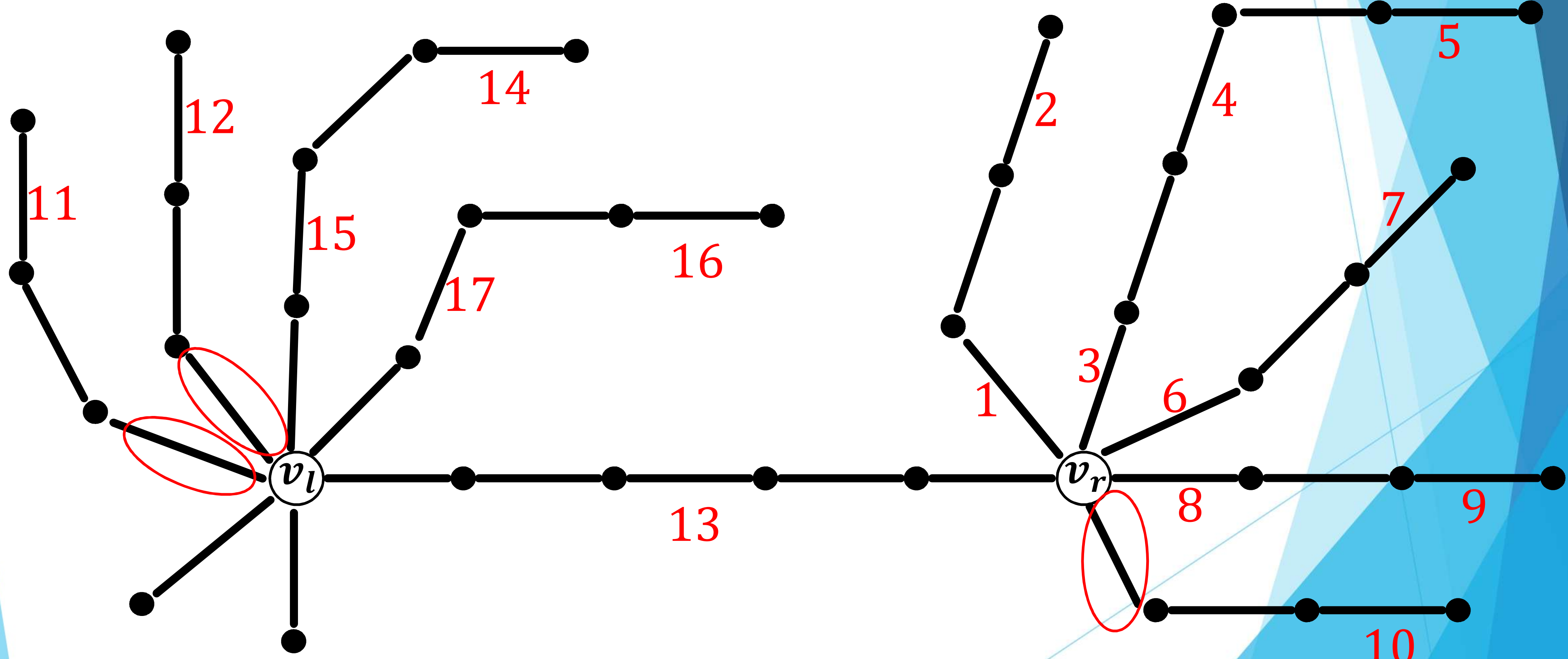


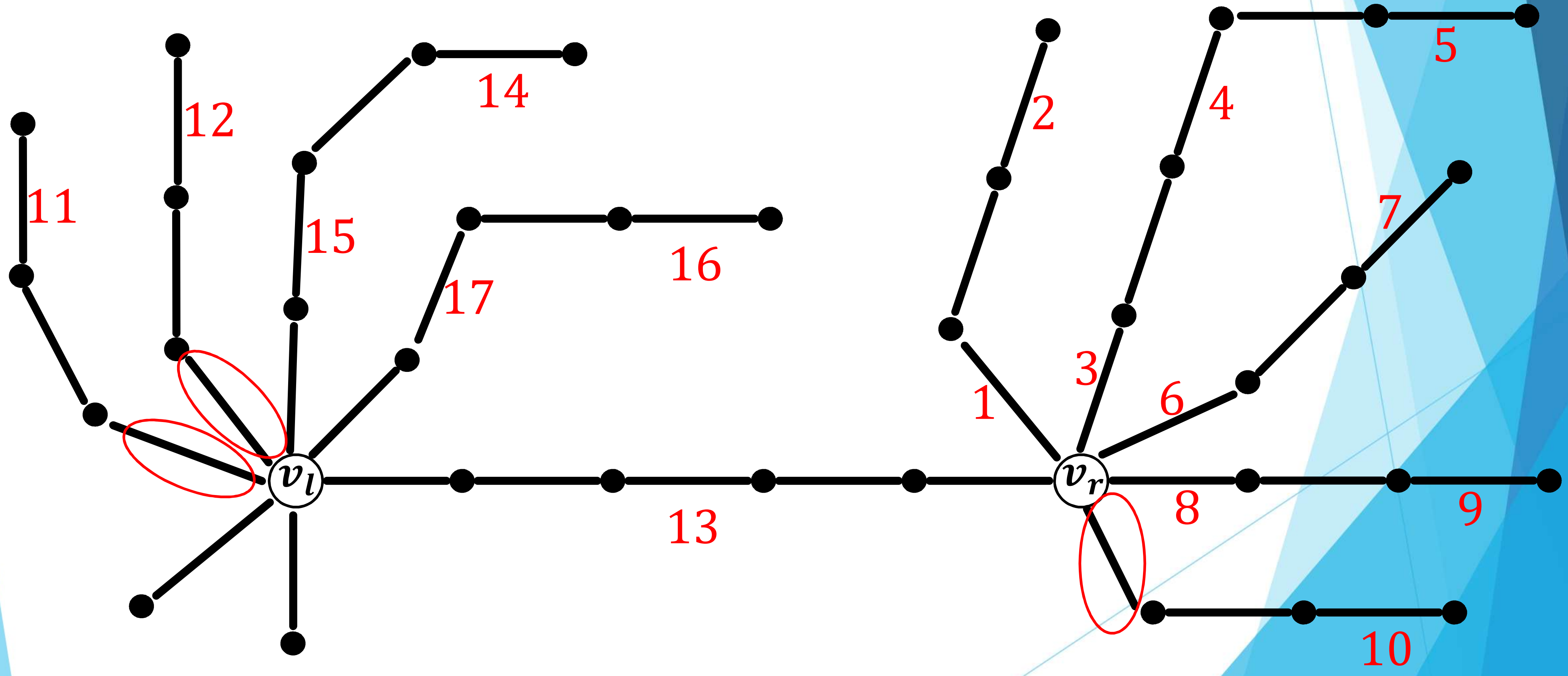
Step 3. If $s \geq 4$, label the edges of P^{core}



Step 4. If $d \geq 1$, label the odd edges of the even paths in L



Step 5. If $t \geq 1$, for $i \in [t]$, label the paths of length one in L



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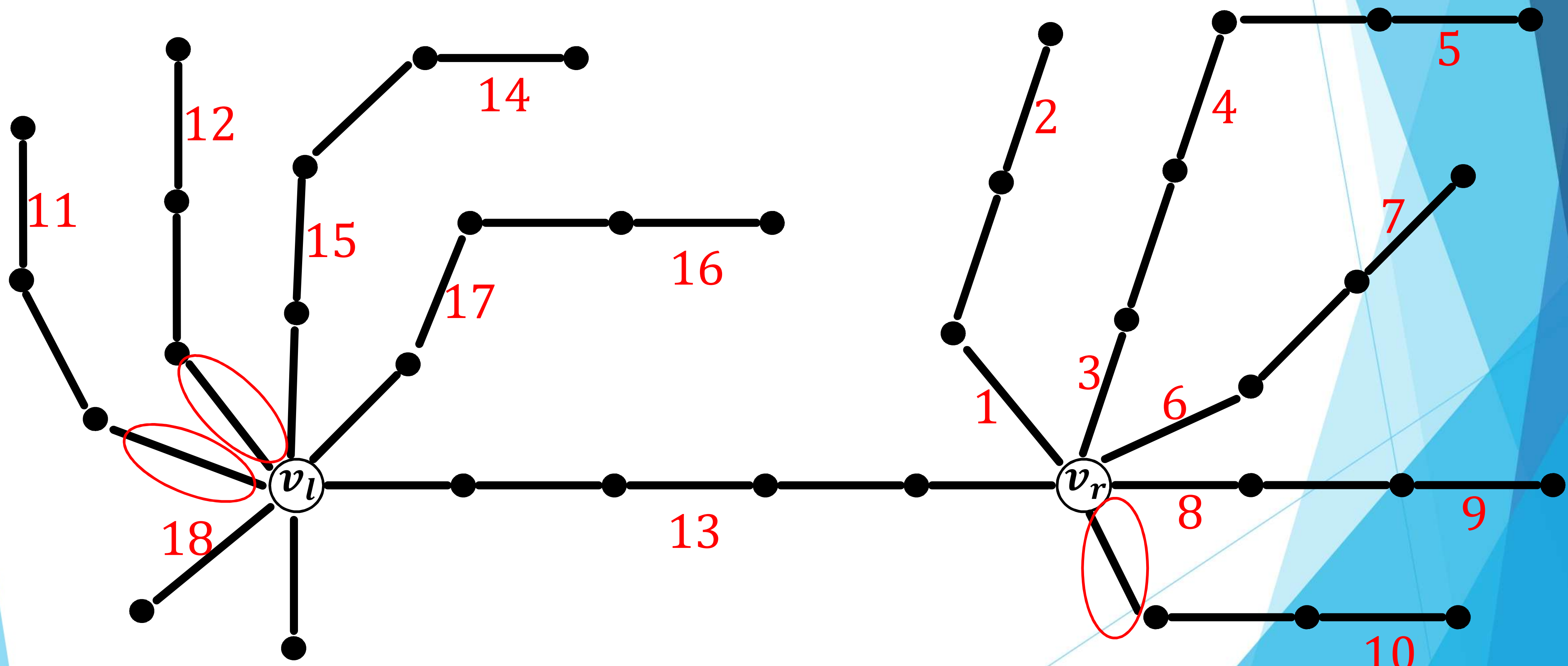

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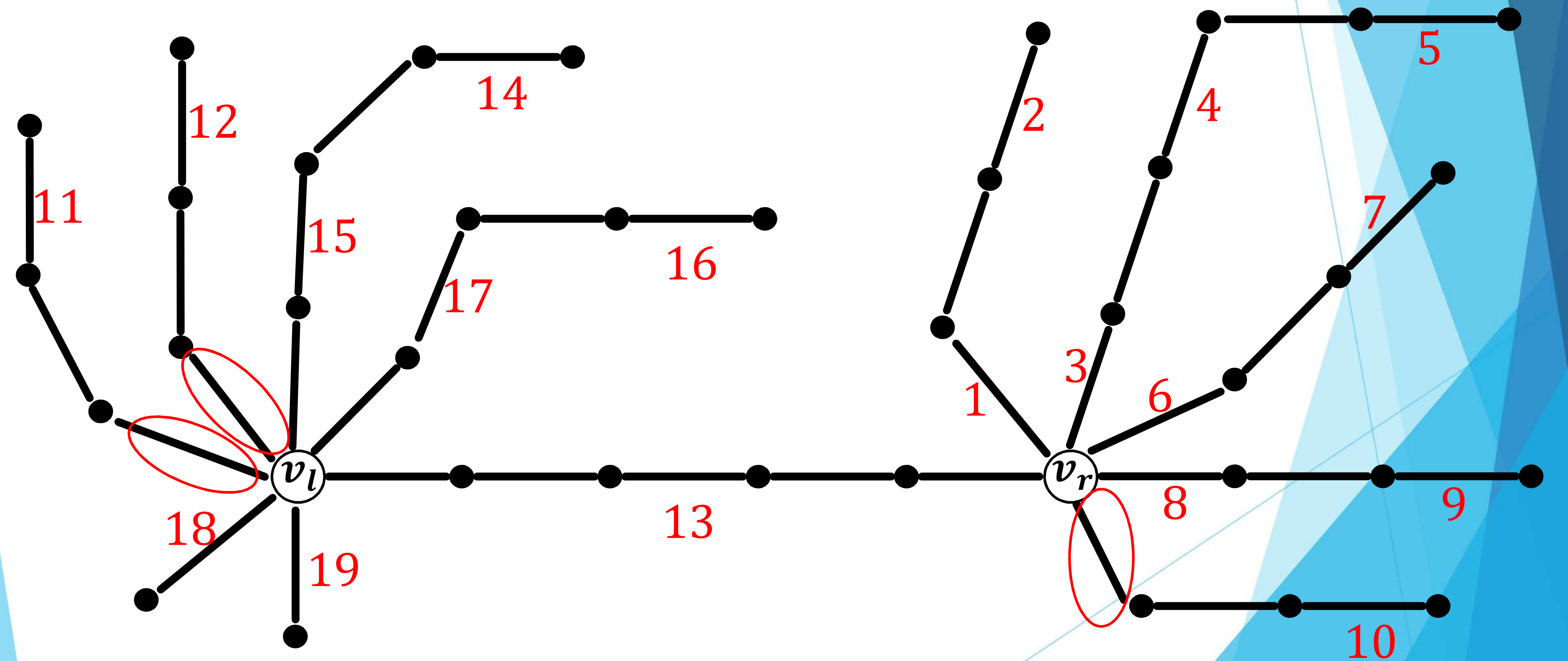
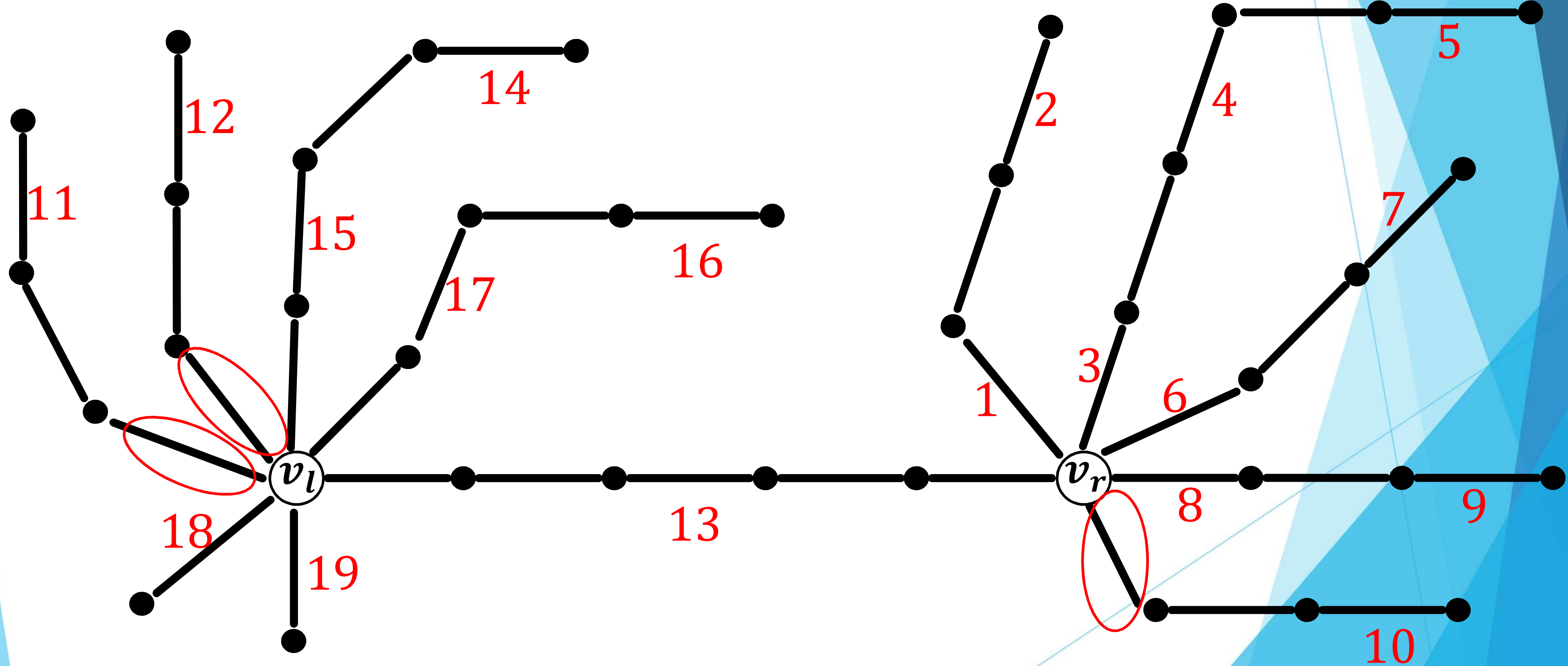


Step 6. label the even edges of the odd paths in R



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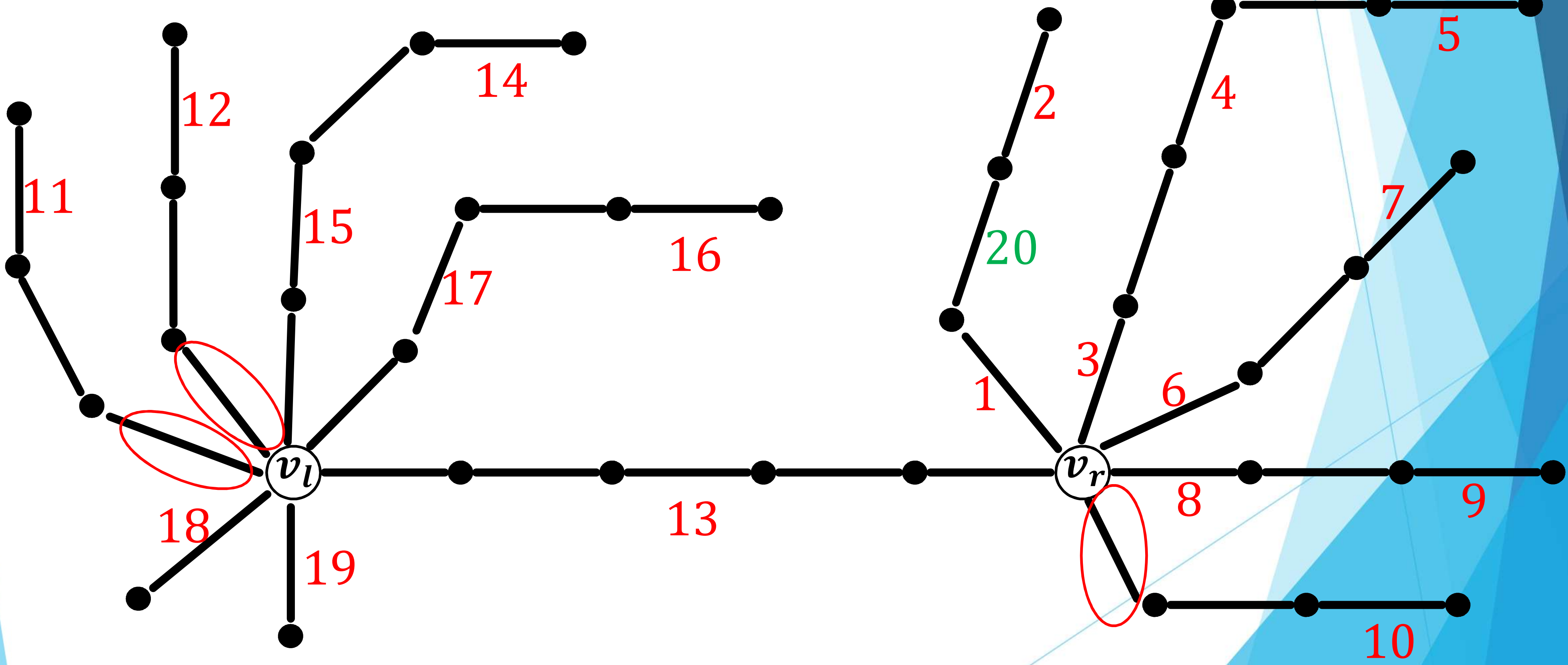
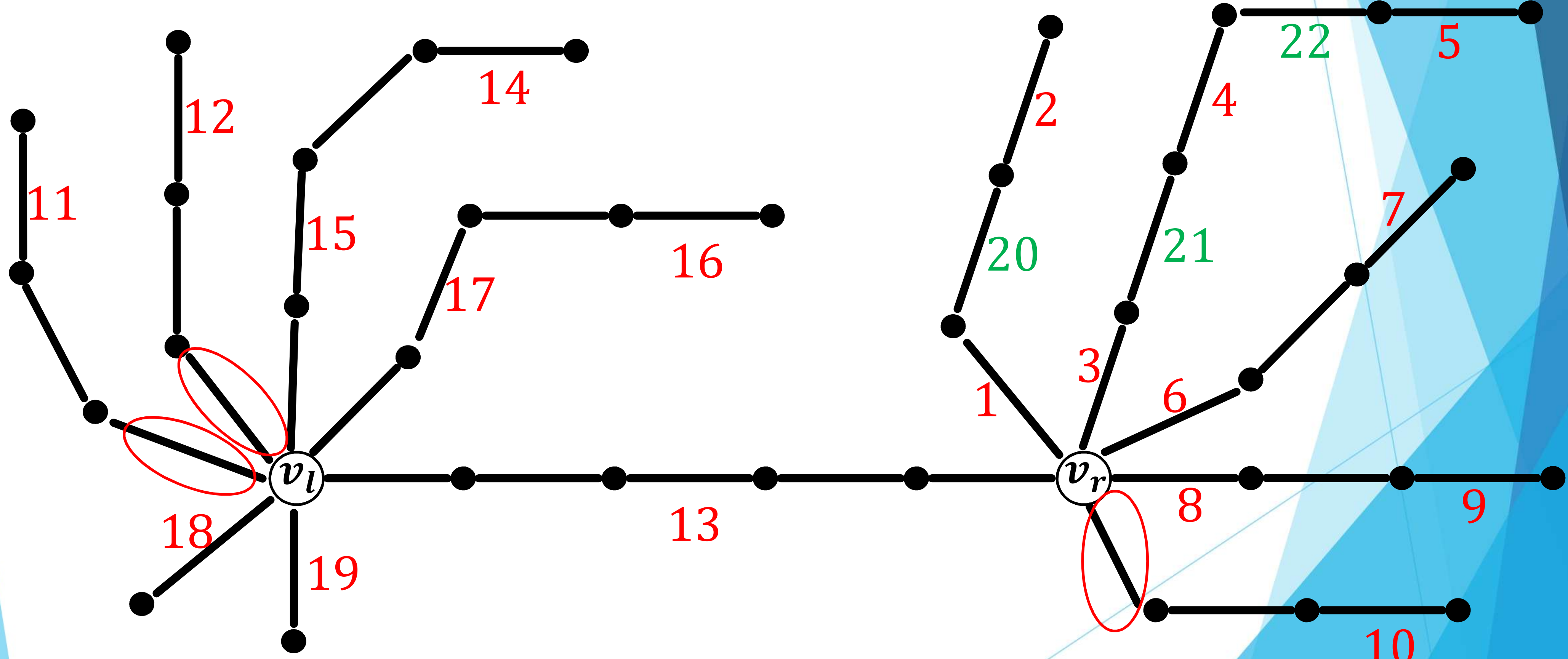


Step 6. label the even edges of the odd paths in R



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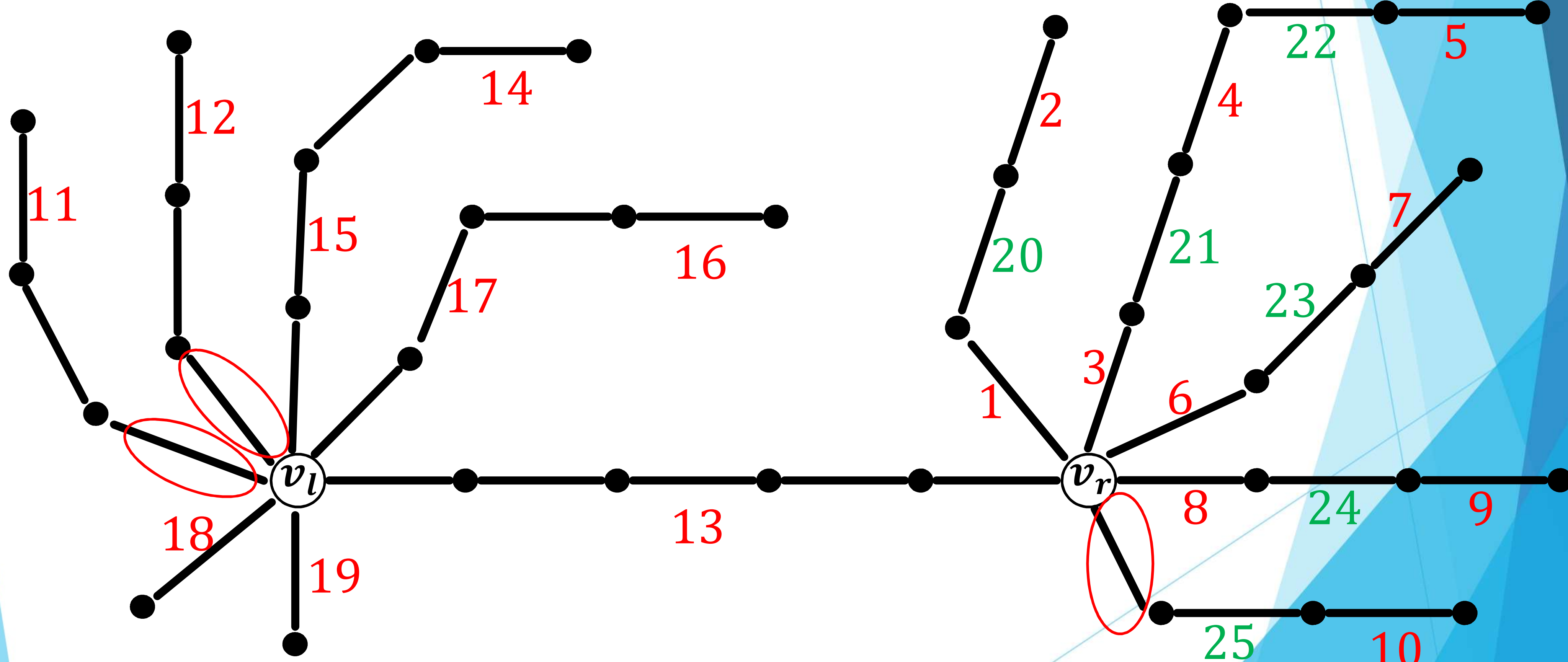
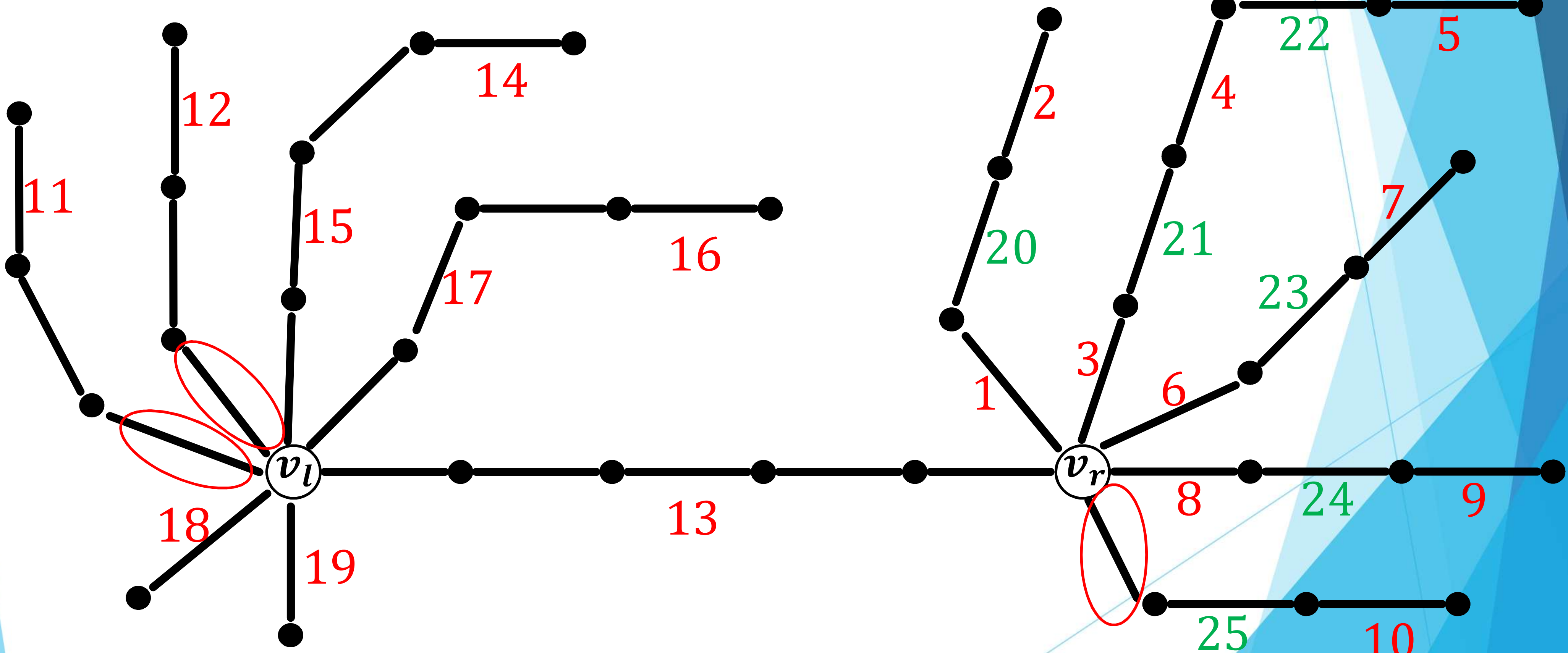


Step 7. If $c \geq 1$, label the even edges of the odd paths in L



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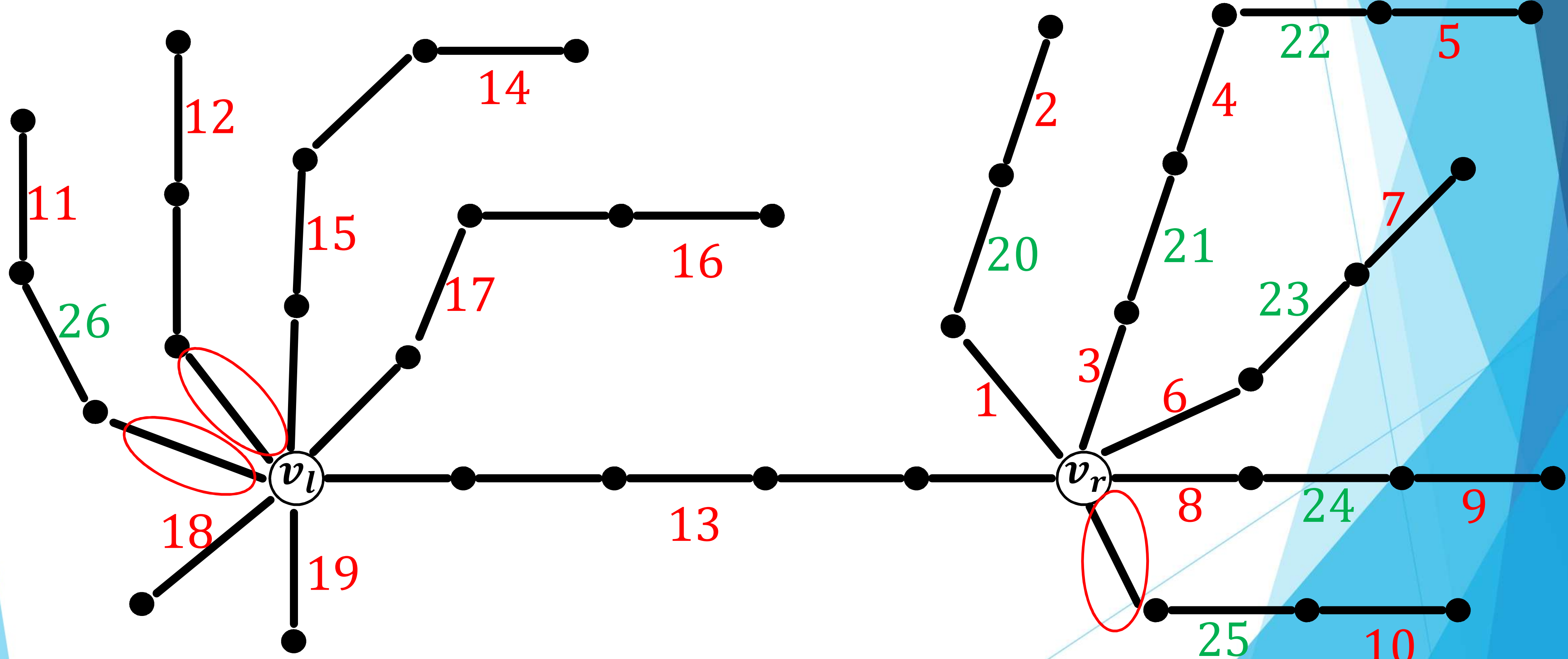

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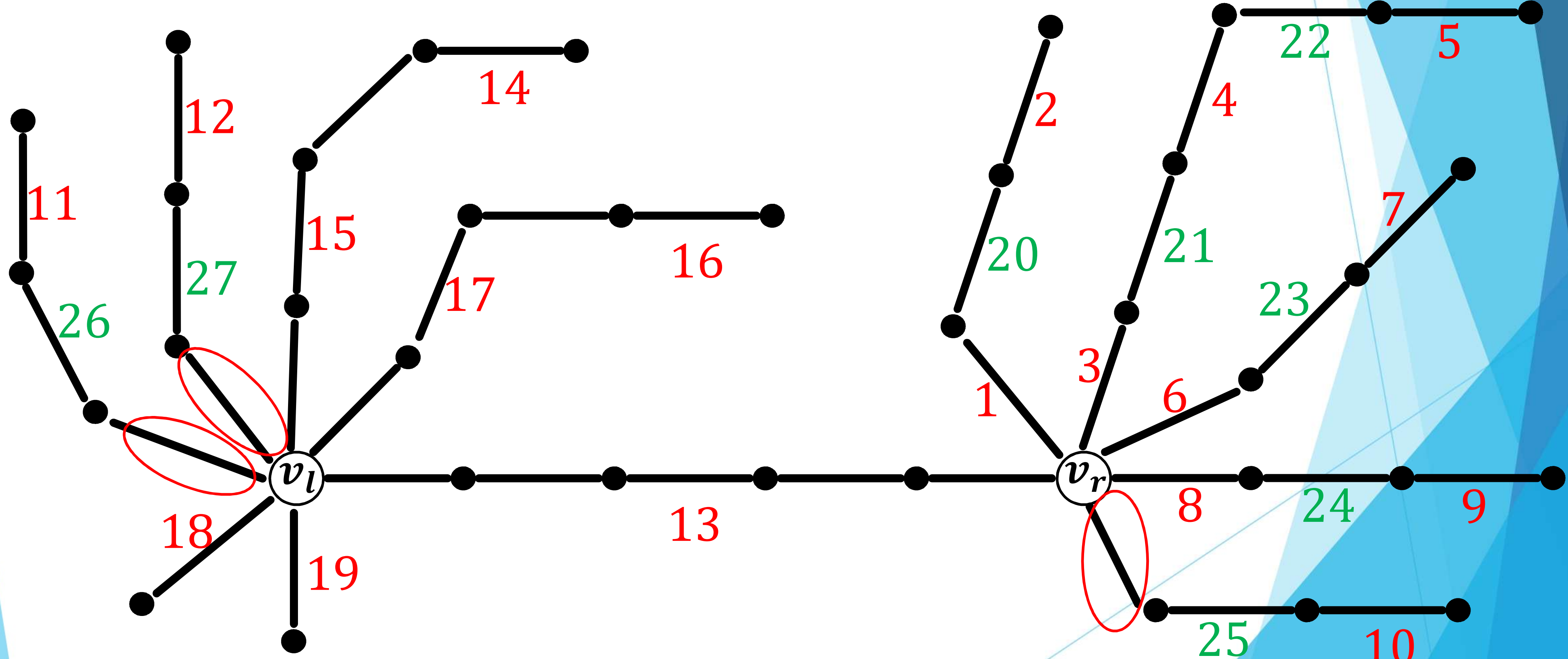
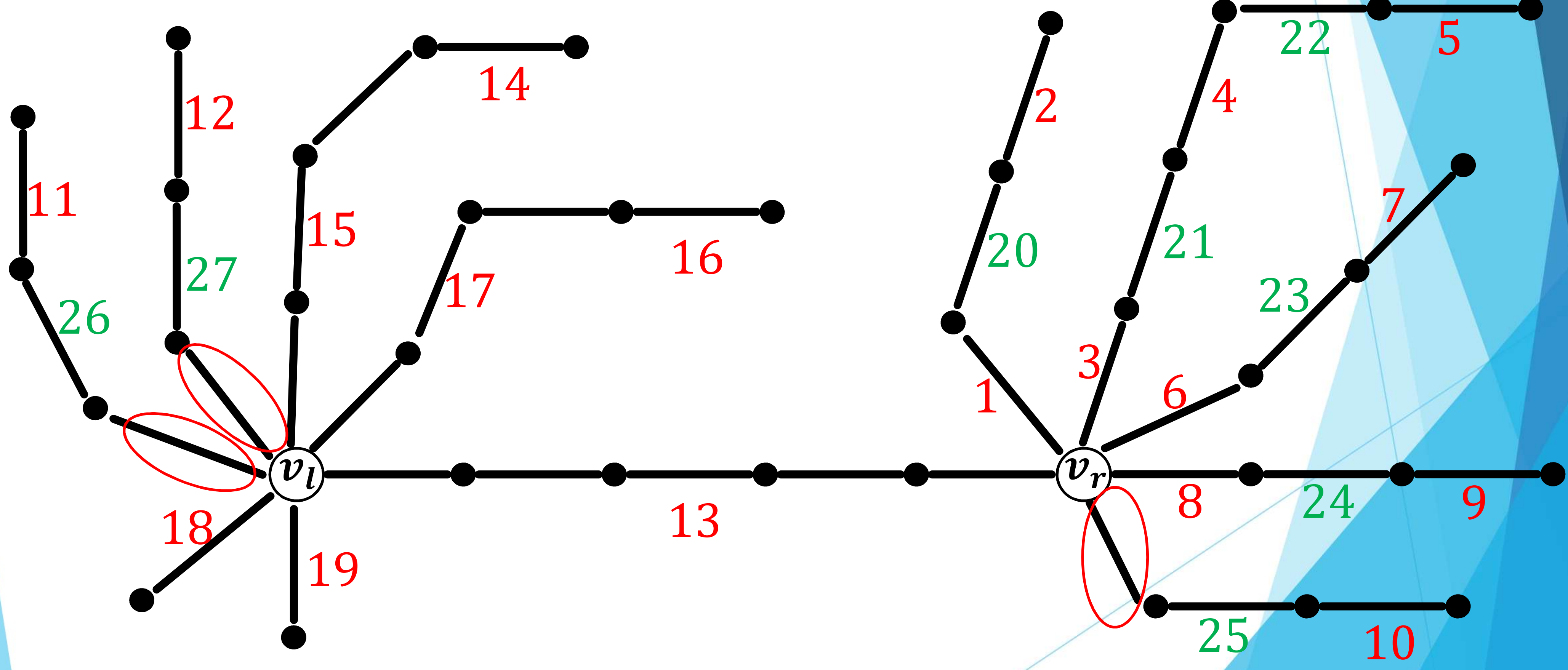


Step 8. If $s \geq 2$, label the edges in P^{core}



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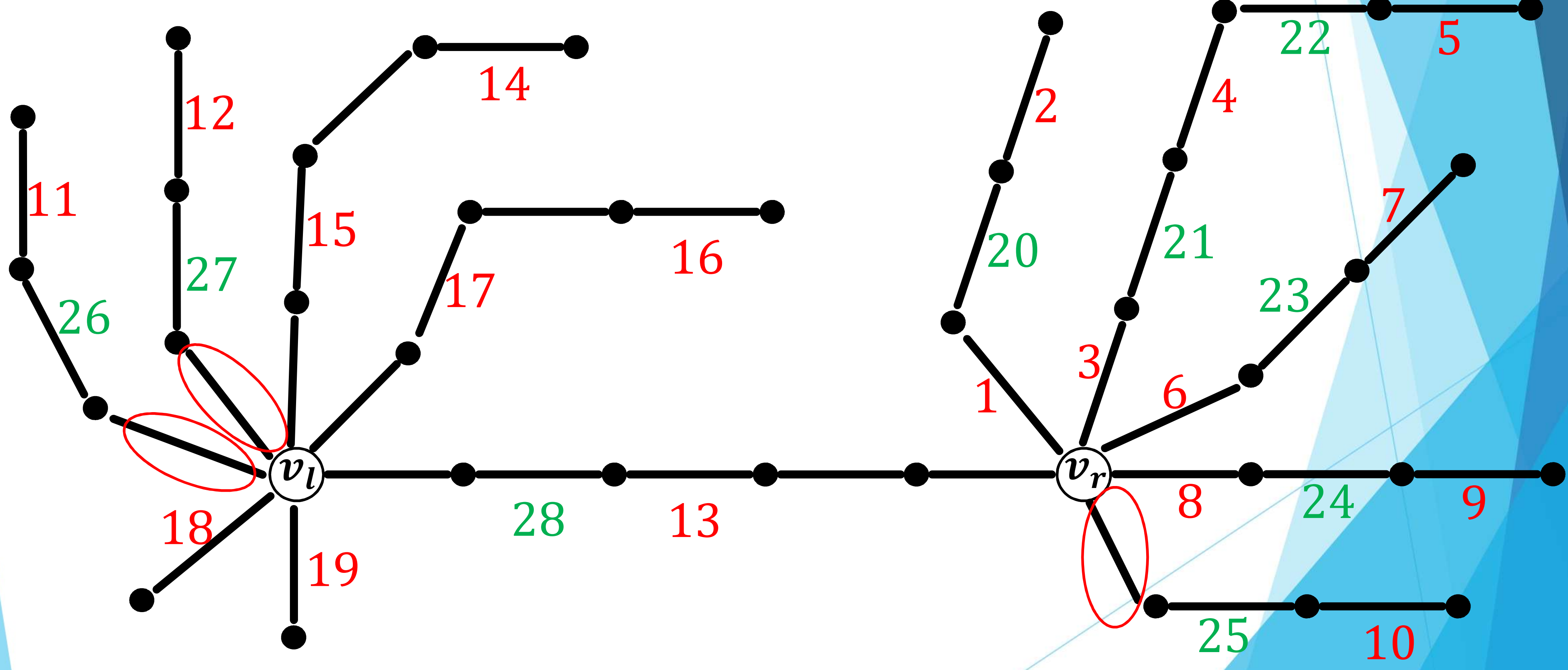
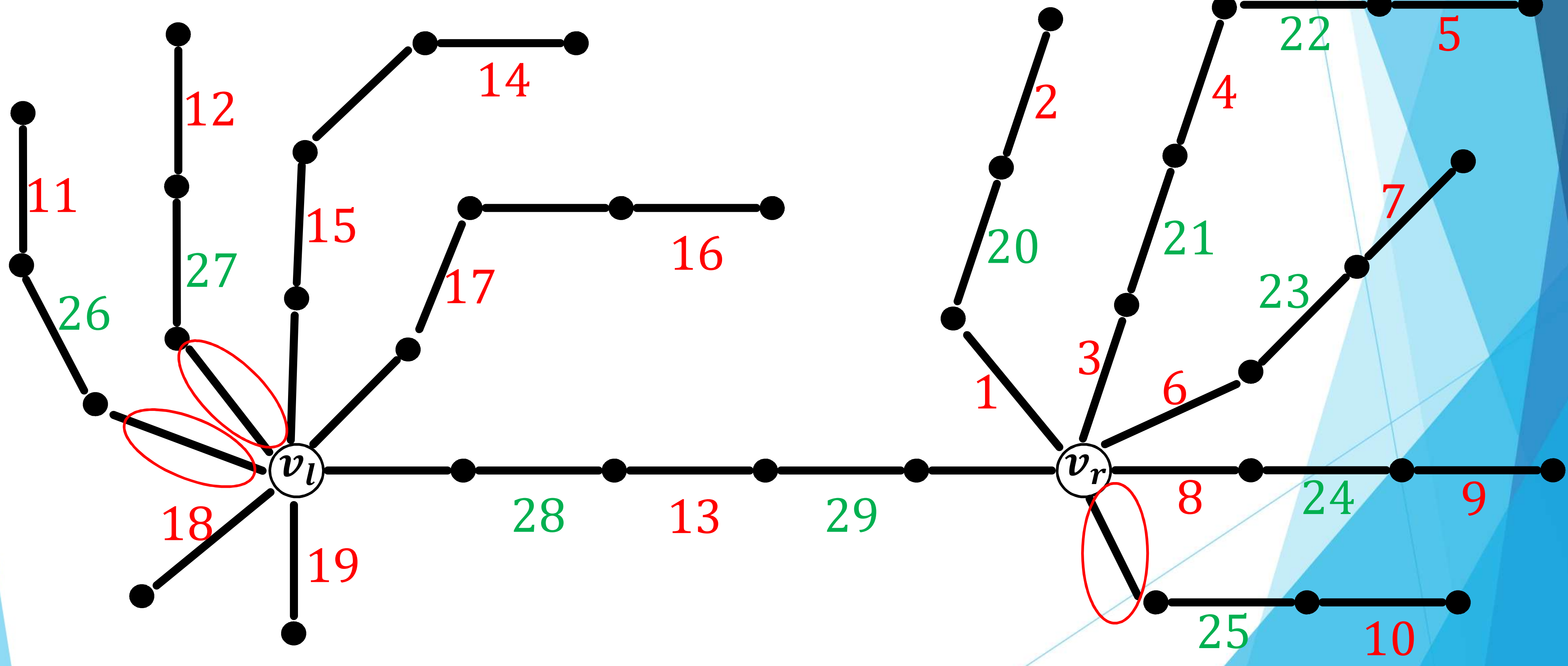
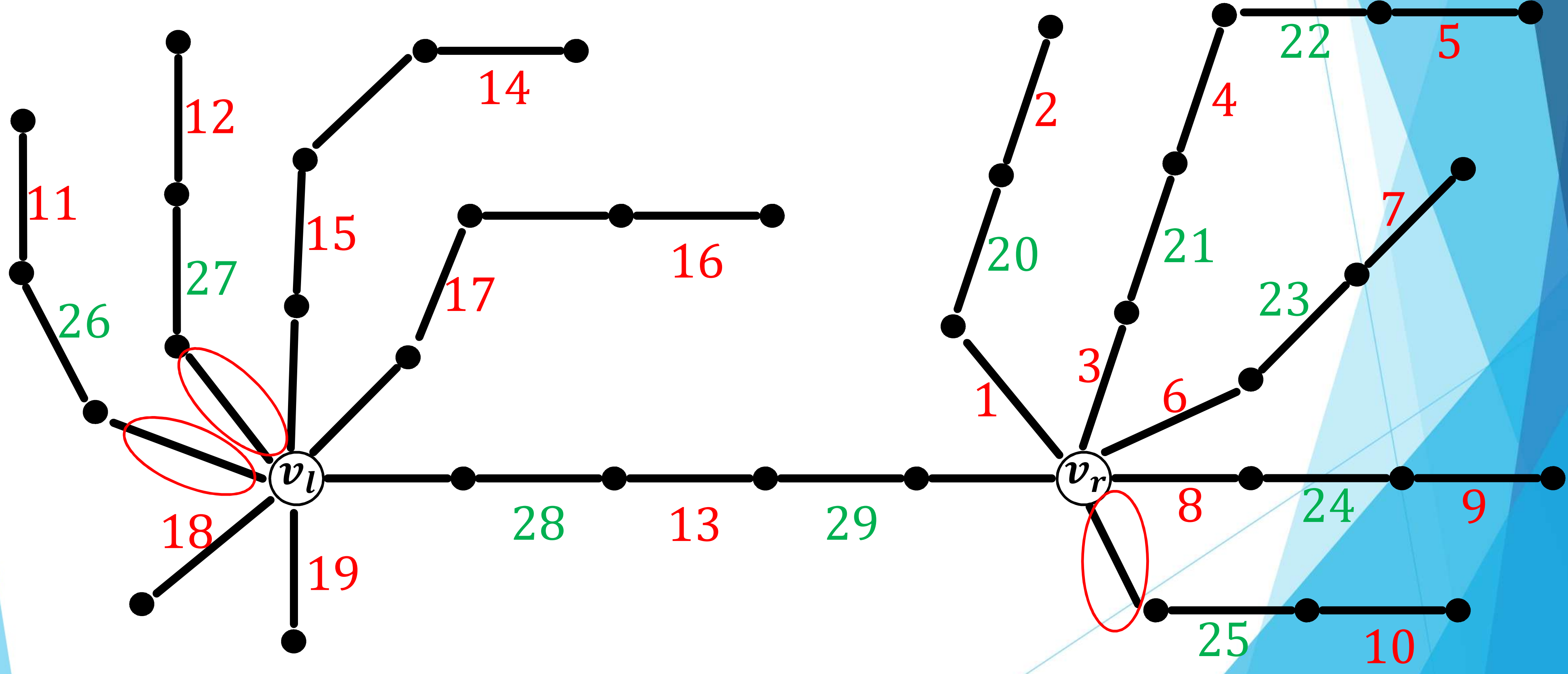


Step 8. If $s \geq 2$, label the edges in P^{core}



Step 9. If $d \geq 1$, label the even edges of the even paths in L



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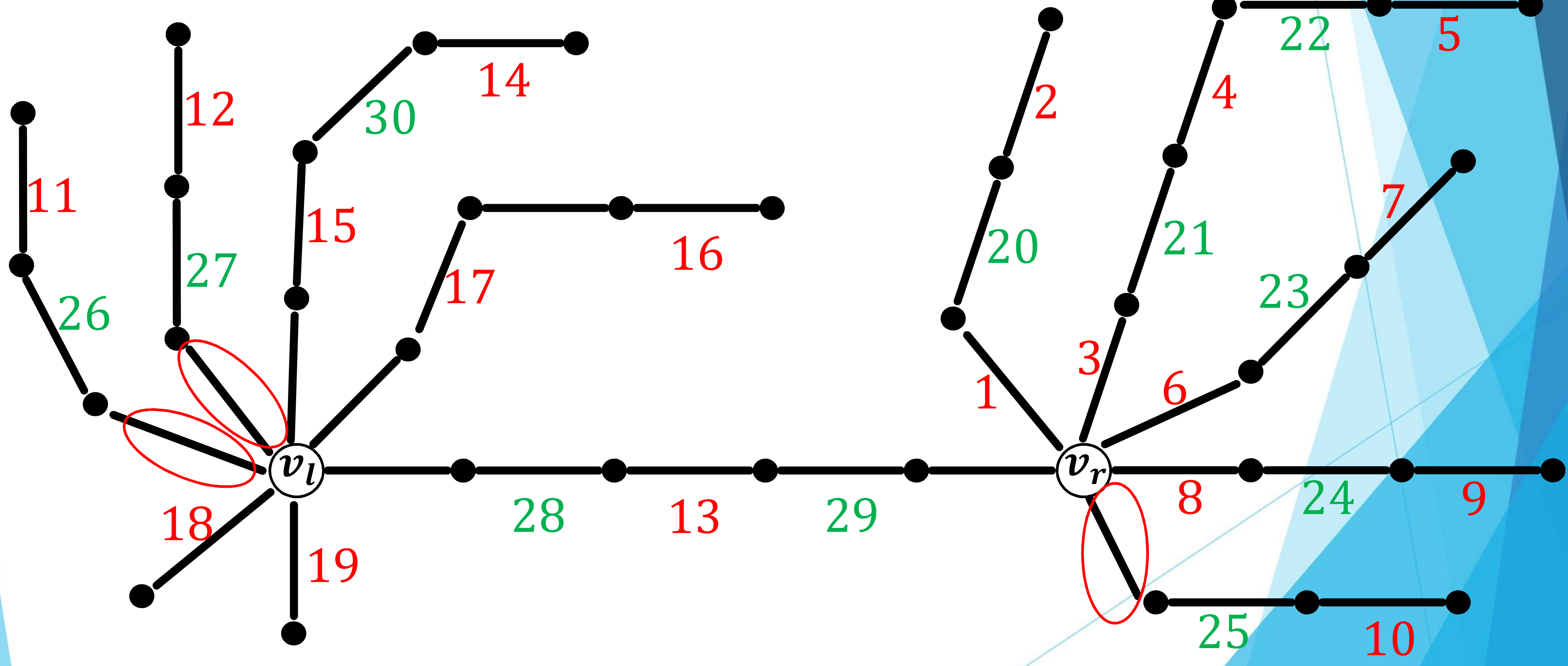

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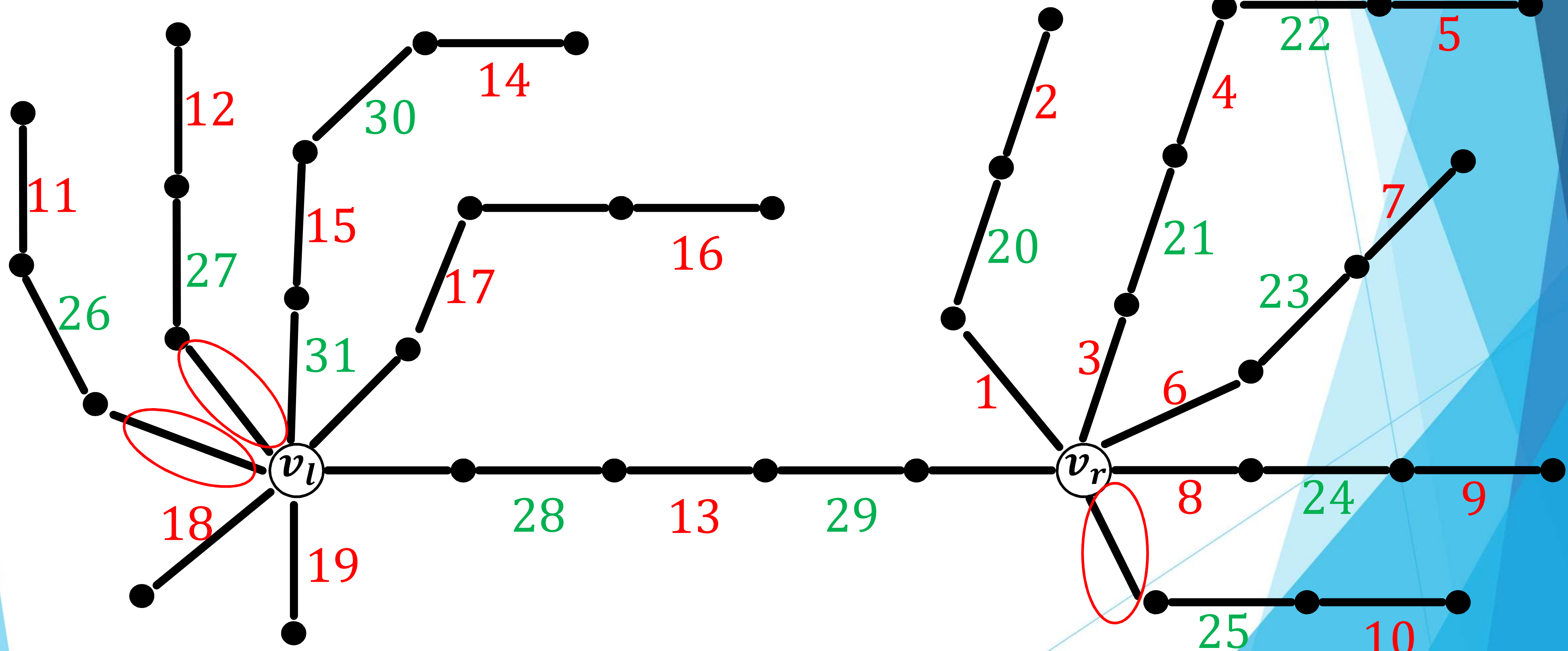

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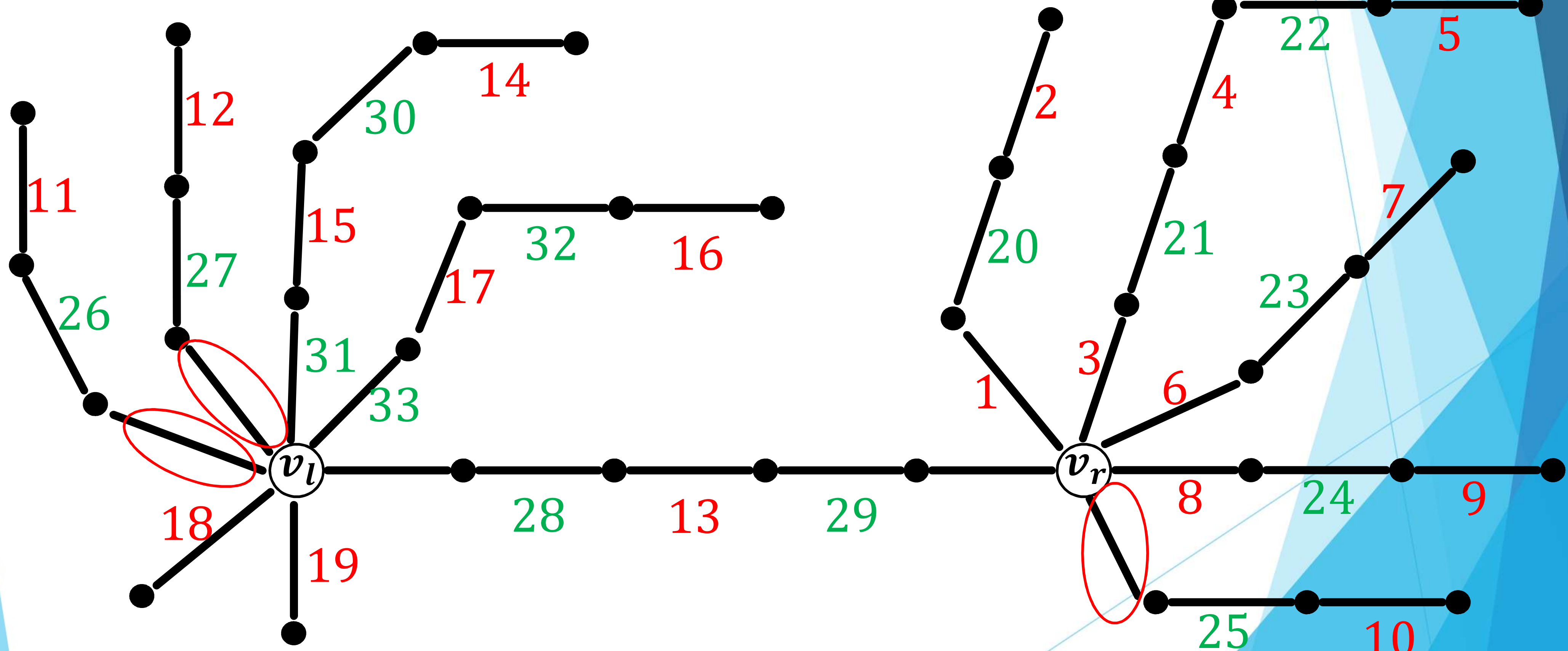
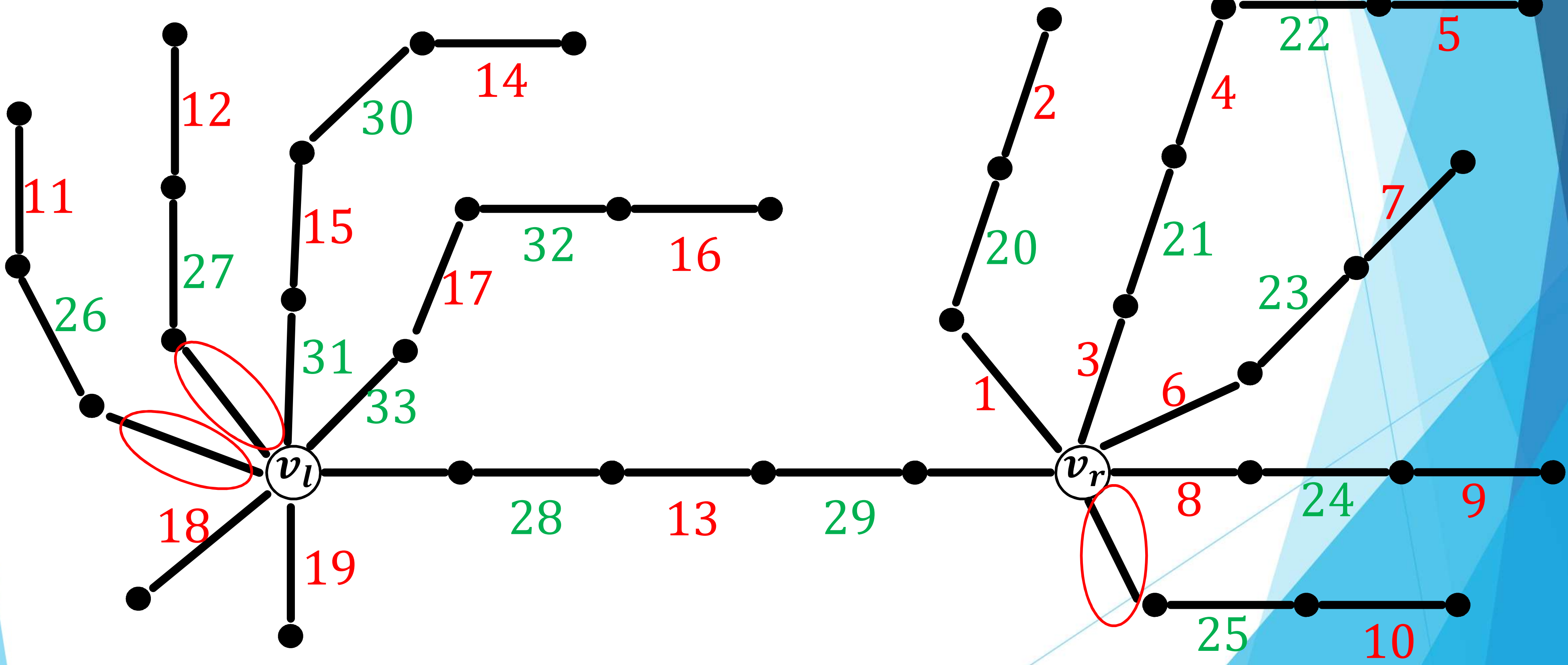


Step 10. Label the edge $e_{a,1}^{r,odd}$



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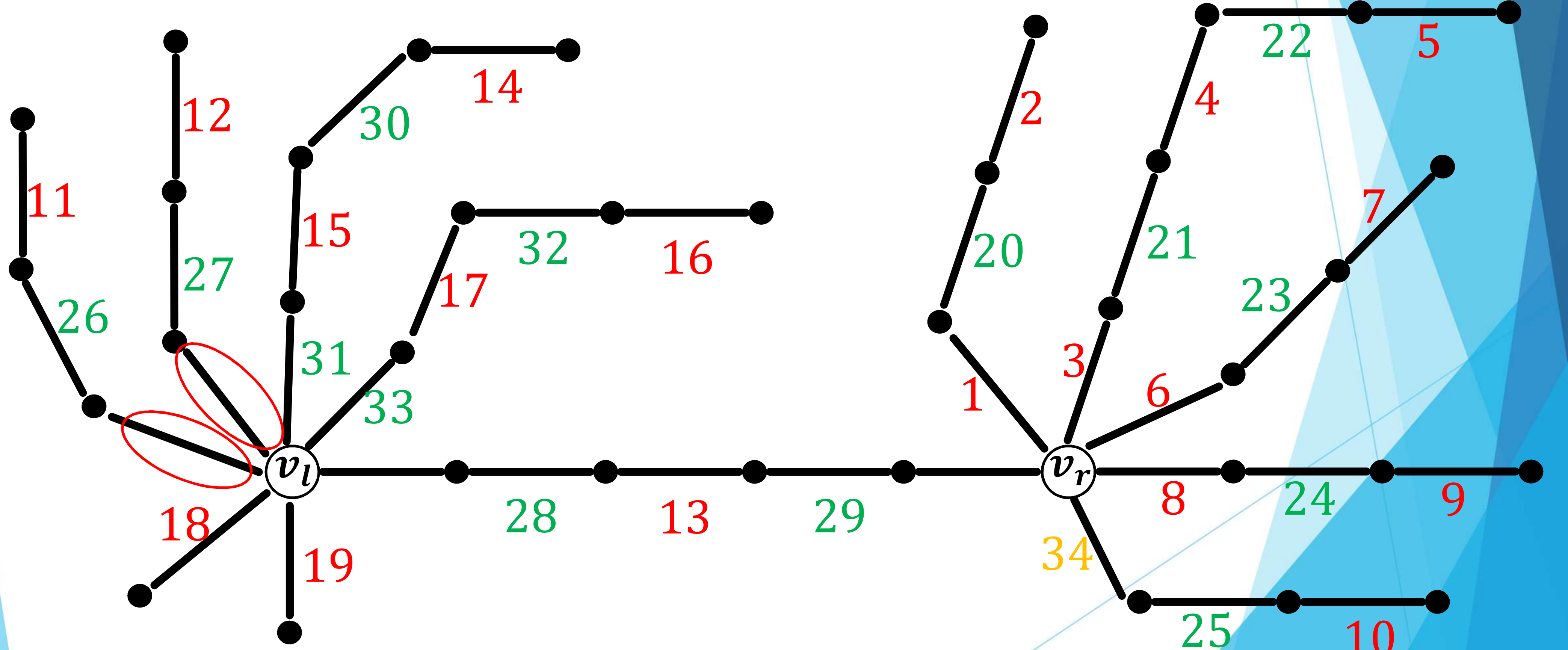
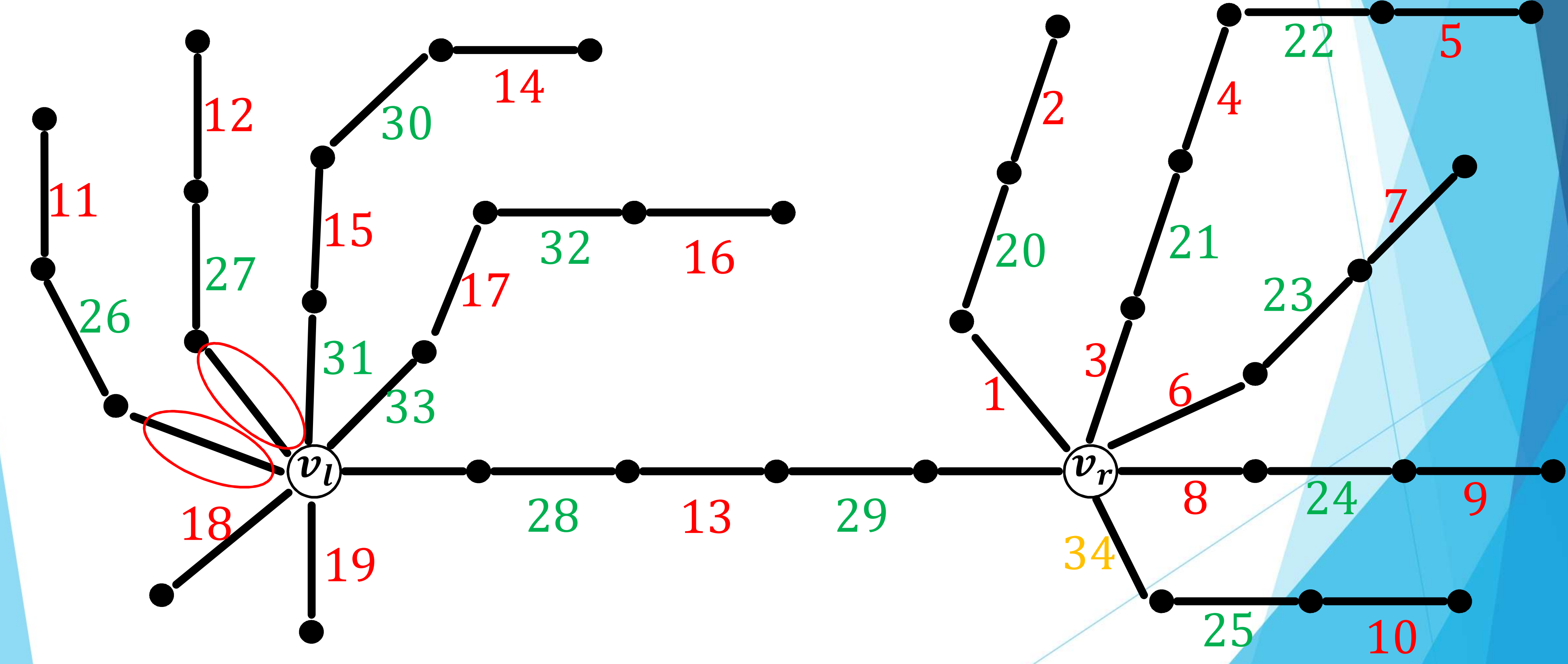


Step 11. If $c \geq 1$, for $i \in [c]$, label the edges $e^{l,odd}_{i,2w_i+1}$



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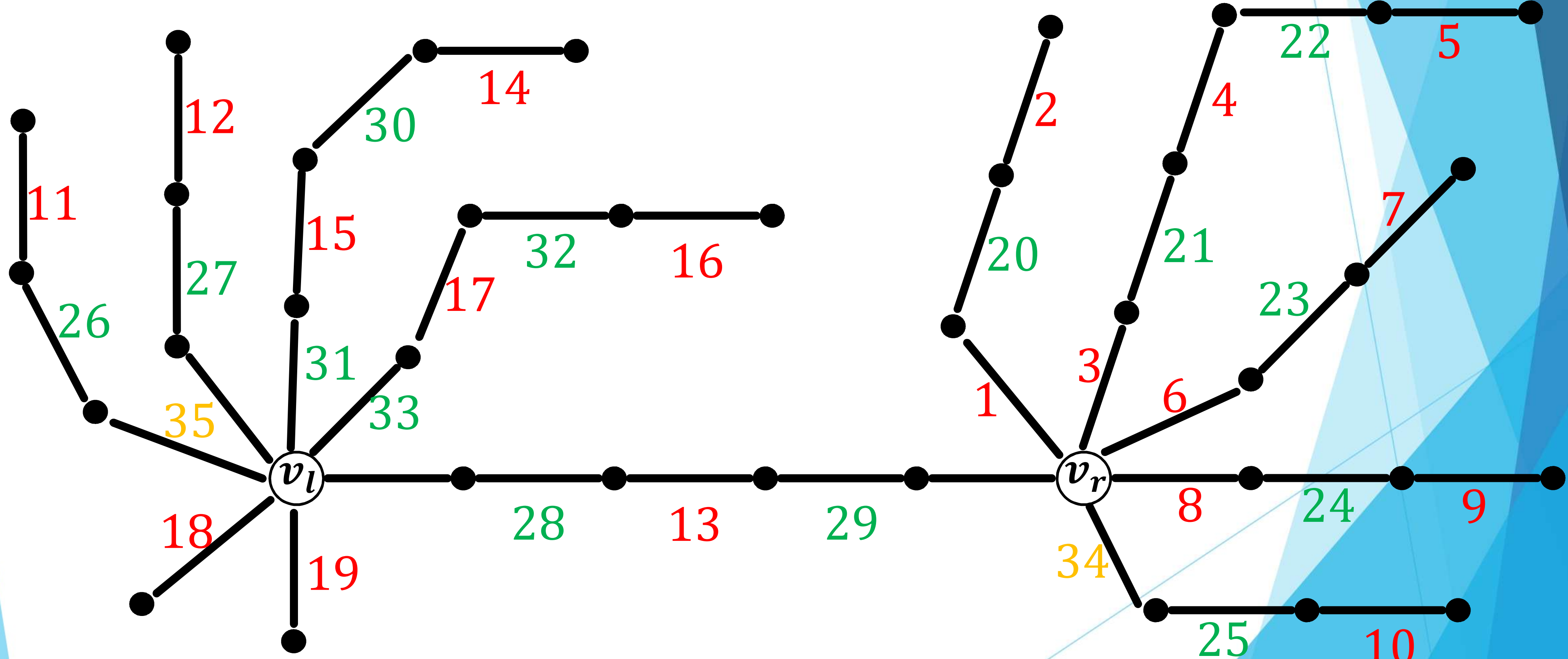

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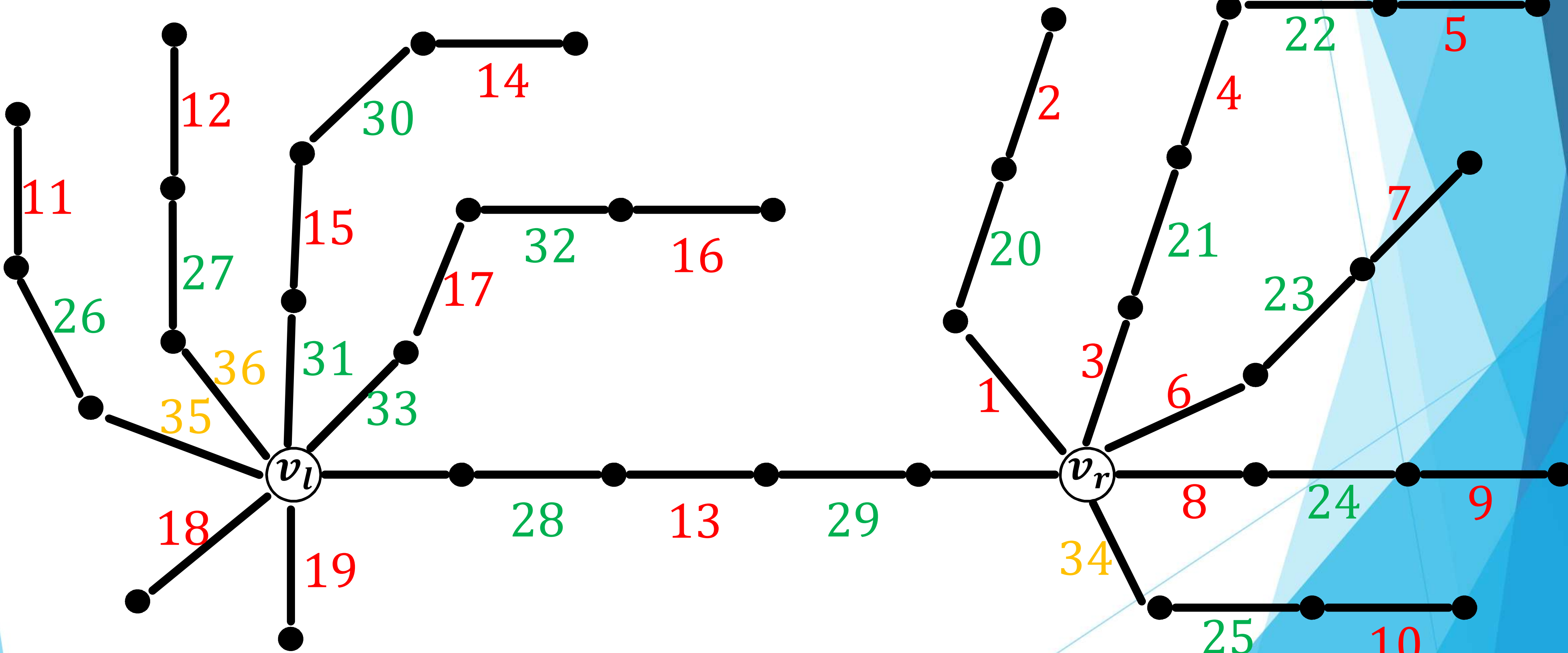
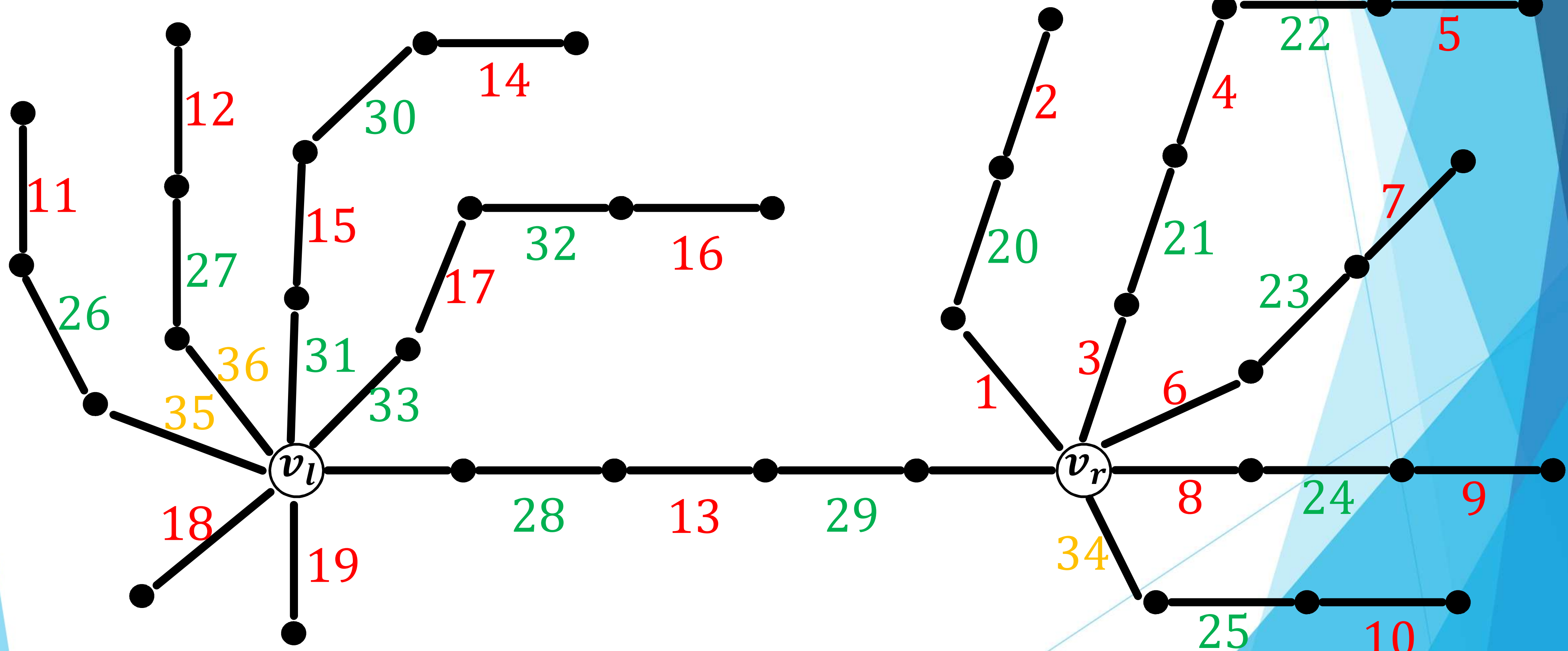


Step 12. Label the remaining edges in P^{core}



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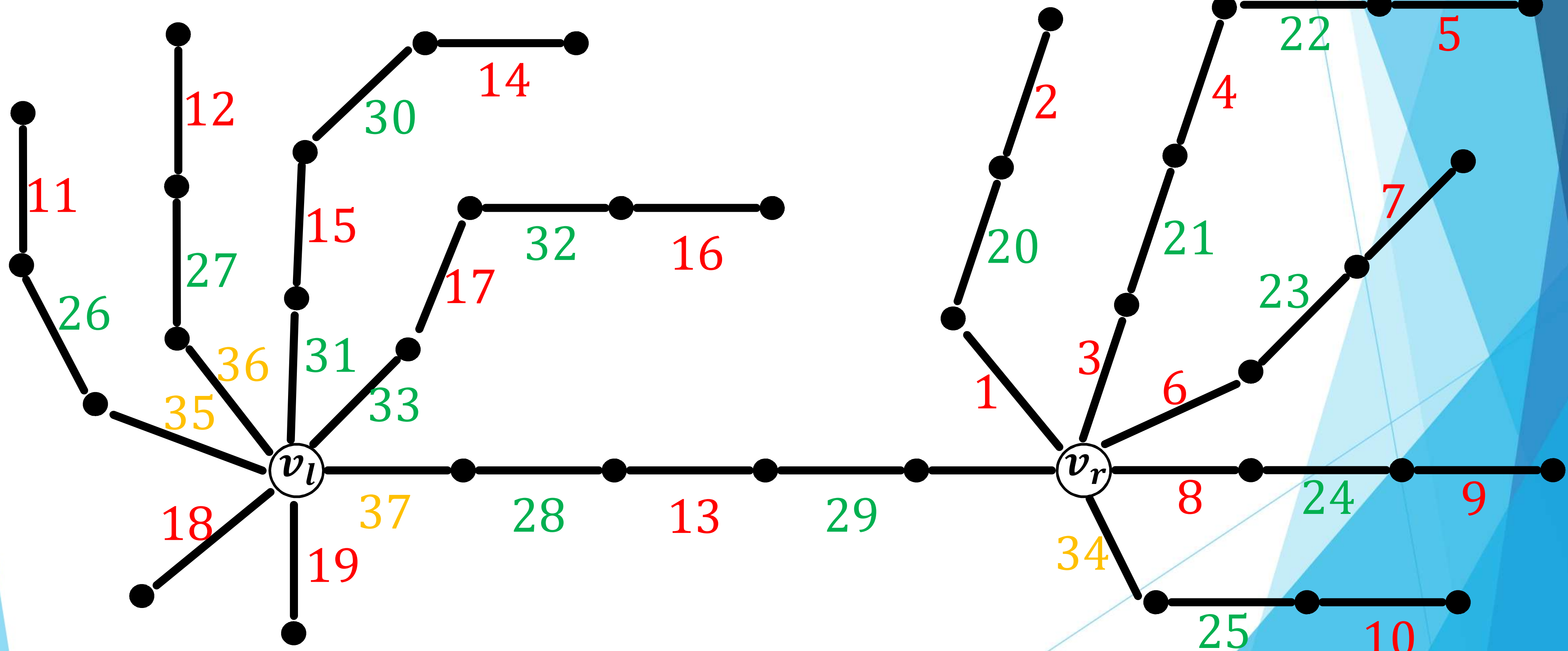

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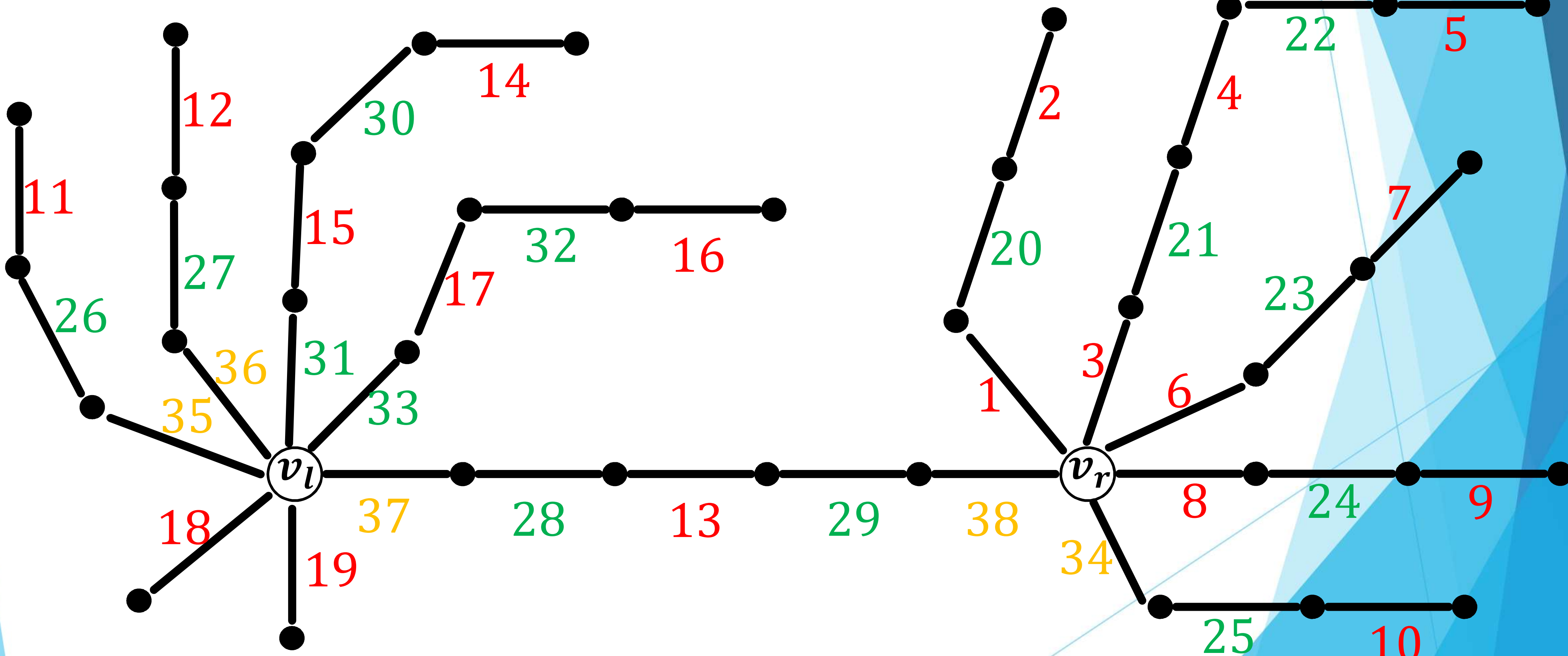
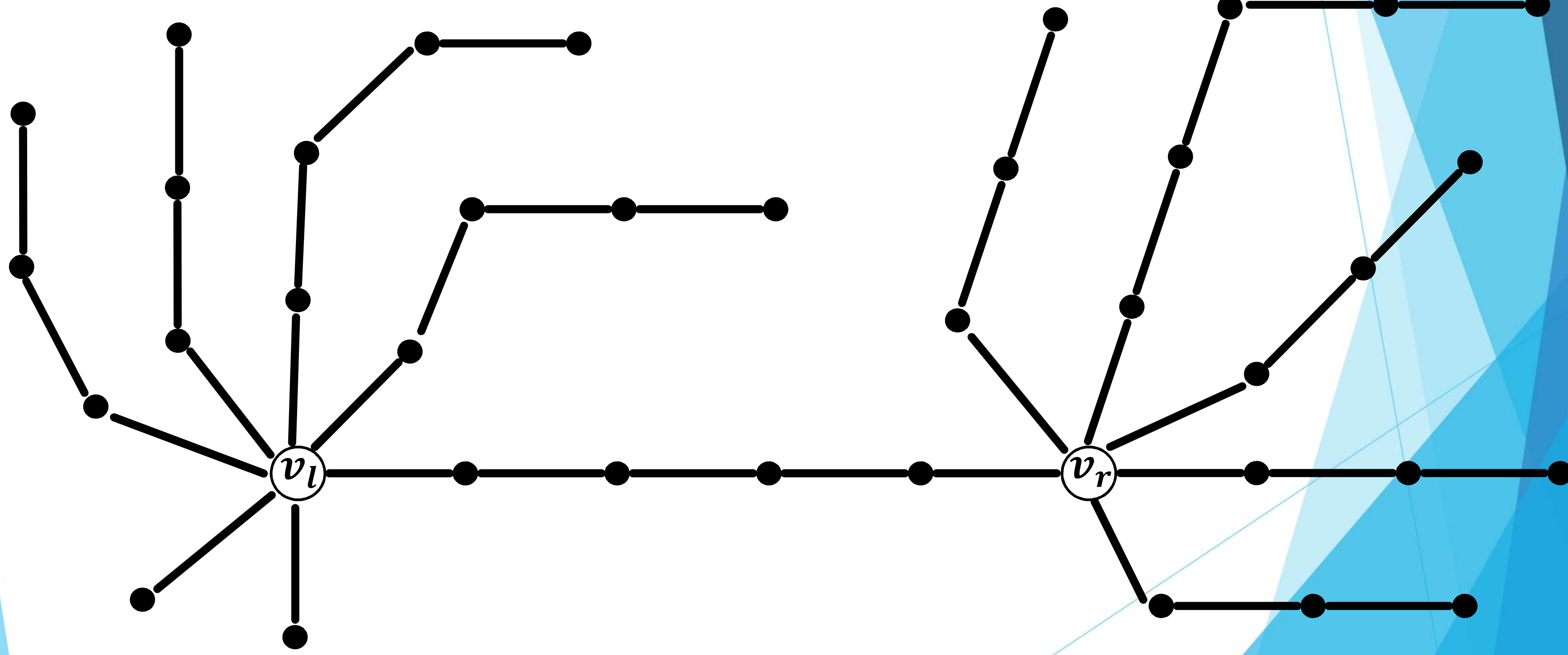


Thank you for your attention!!



v_l
v_r