Set distance labelings on edges of diamond necklaces

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Introduction

Notations

- Let G be a graph, n be a positive integer and δ_1, δ_2 be non-negative integers.
- Denote $[k] = \{0, 1, ..., k\}$ and $\binom{[k]}{n}$ the collection of all n-element subsets of $[k]$.
- Let A and B be two sets of numbers. Define $\|A - B\| = \min\{|a - b| : a \in A, b \in B\}$.

Definition

An $L_e^{(n)}(\delta_1, \delta_2)$-*labeling* ϕ of a graph G is a function $\phi : E(G) \rightarrow \binom{[k]}{n}$ such that $\|\phi(e_1) - \phi(e_2)\| \geq \delta_i$ whenever the distance between e_1 and e_2 is i in G for $i = 1, 2$, for some k. The value of k is called the *edge span* of ϕ.
The smallest edge span over all $L_e^{(n)}(\delta_1, \delta_2)$-labelings is called the $L_e^{(n)}(\delta_1, \delta_2)$-*labeling number* of G and is denoted by $\lambda_e^{(n)}(G; \delta_1, \delta_2)$.

Introduction

In this article, we will consider the case where $(\delta_1, \delta_2) = (2, 1)$. The corresponding labeling is denoted by $\lambda_e^{(n)}(G)$ for short.

In the following study, we first consider $n = 2$. Once, the problem is solved, it is easy to extend to $n \geq 2$.

In the previous study, we have defined an analogous vertex version of this kind of labeling, called the $L^{(n)}(\delta_1, \delta_2)$-labeling.

The corresponding labeling is denoted by $\lambda^{(n)}(G; \delta_1, \delta_2)$, where $\lambda^{(n)}(G)$ is short for $\lambda^{(n)}(G; 2, 1)$.

Example. An $L_e^{(2)}(2,1)$-labeling on C_4

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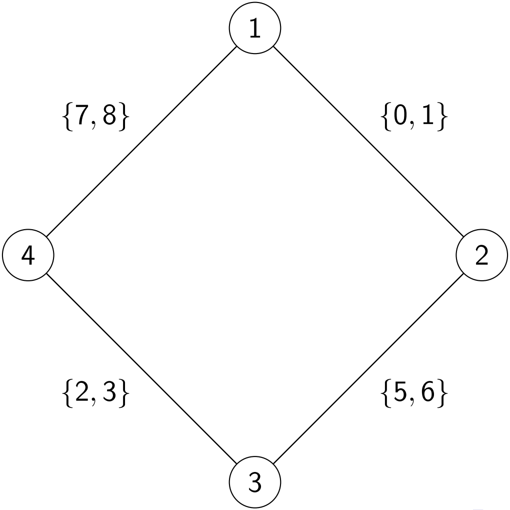


Line Graph

Given a gaph $G = (V, E)$, the *line graph* $L(G)$ of the graph G, is the graph with $V(L(G)) = E(G)$ and two edges e and f are adjacent in G if and only if the corresponding vertices v_e and v_f are adjacent in $L(G)$.

Line Graph

$$L_e^{(n)}(\delta_1, \delta_2)\text{-labeling on } G \longleftrightarrow L^{(n)}(\delta_1, \delta_2)\text{-labeling on } L(G)$$

Line Graph

Let P_t and C_t be path and cycle of order $t \geq 3$, respectively. Then we have
$L(P_t) = P_{t-1}$ and $L(C_t) = C_t$.

Hence, $\lambda_e^{(n)}(P_t) = \lambda^{(n)}(P_{t-1})$ and $\lambda_e^{(n)}(C_t) = \lambda^{(n)}(C_t)$.

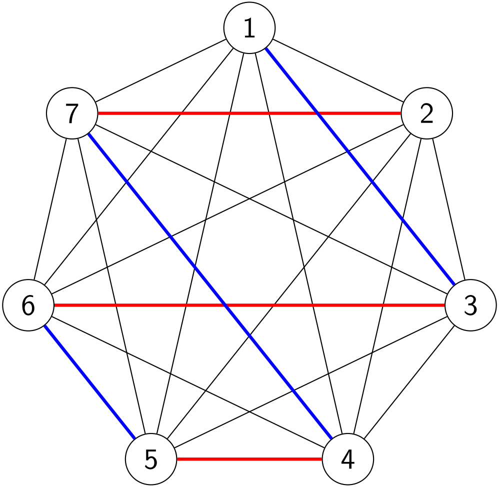
Edge Coloring on K_m

Theorem

$$\chi'(K_m) = \begin{cases} m-1 & \text{if } m \text{ is even,} \\ m & \text{if } m \text{ is odd.} \end{cases}$$

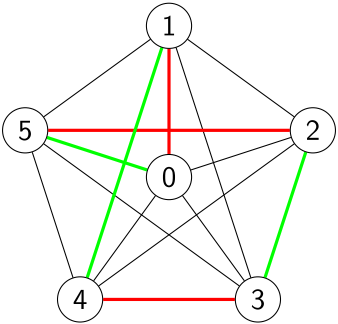
Edge Coloring on K_7

$\chi'(K_7) = 7$



Edge Coloring on K_6

$\chi'(K_6) = 5$



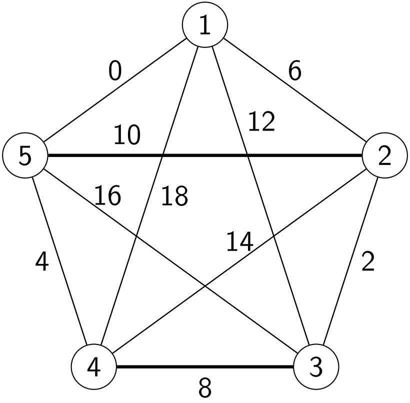
Edge Labeling on K_m

Theorem

$$\lambda_e^{(n)}(K_m) = \begin{cases} n\epsilon - 1 & \text{if } m = 2 \text{ or } m \geq 5, \\ n\epsilon + 1 & \text{if } m = 3, 4, \end{cases}$$

where $\epsilon = \binom{m}{2}$ the size of K_m.

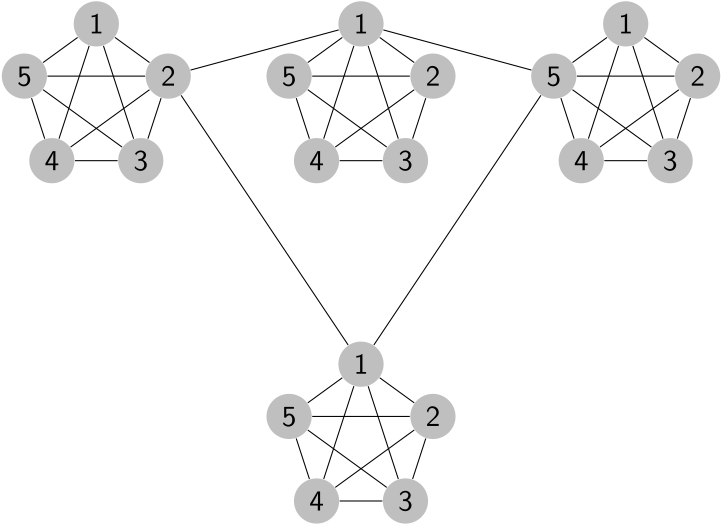
$\lambda_e^{(2)}(K_5) = 19$



Diamond Necklaces

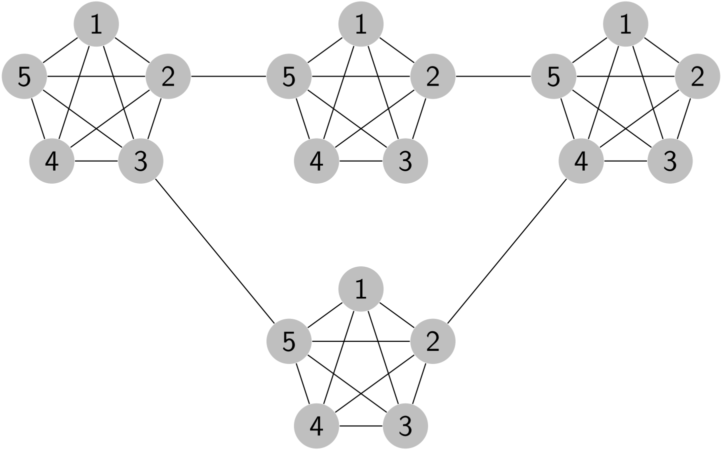
$D_1(tK_m)$ and $D_2(tK_m)$

Diamond Necklaces



$D_1(4K_5)$

Diamond Necklaces



$D_2(4K_5)$

Cycle

From the observation above, we know that $\lambda_e^{(2)}(C_t) = \lambda^{(2)}(C_t)$. Thus, by previous results on $\lambda^{(2)}(C_t)$, we have

Theorem

$$\lambda_e^{(2)}(C_t) = \begin{cases} 9 & t = 5, \\ 8 & t = 4, 7, 10, 13, \\ 7 & \textit{otherwise}. \end{cases}$$

$\lambda_e^{(2)}(D_1(tK_m))$

$\lambda_e^{(2)}(D_1(tK_m))$, $\lambda_e^{(2)}(K_m)$ and $\lambda_e^{(2)}(C_t)$???

$\lambda_e^{(2)}(D_1(tK_m))$, $\lambda_e^{(2)}(K_m)$ and $\lambda_e^{(2)}(C_t)$???

Take a guess!!!

$\lambda_e^{(2)}(D_1(tK_m))$, $\lambda_e^{(2)}(K_m)$ and $\lambda_e^{(2)}(C_t)$???

Take a guess!!!

$\lambda_e^{(2)}(D_1(tK_m))$ is about , $\eta = \lambda_e^{(2)}(K_m) + \lambda_e^{(2)}(C_t) + 1$.

$D_1(tK_5)$, t even

Theorem

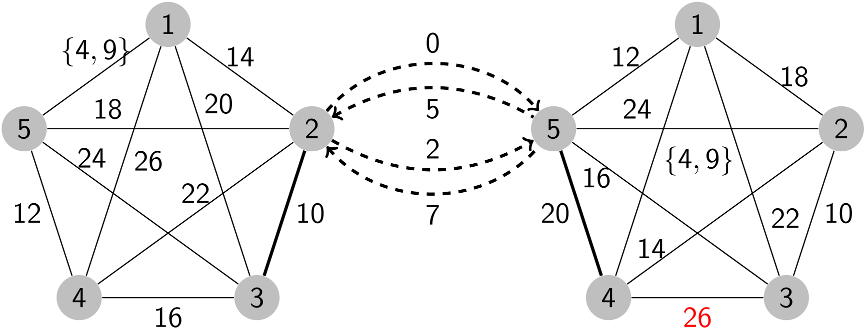
Let $t \geq 4$ be even. Then

$$\lambda_e^{(2)}(D_1(tK_5)) = \begin{cases} 25 & t \equiv 0 \pmod 3, \\ 28 & t = 10, \\ 27 & \textit{otherwise.} \end{cases}$$

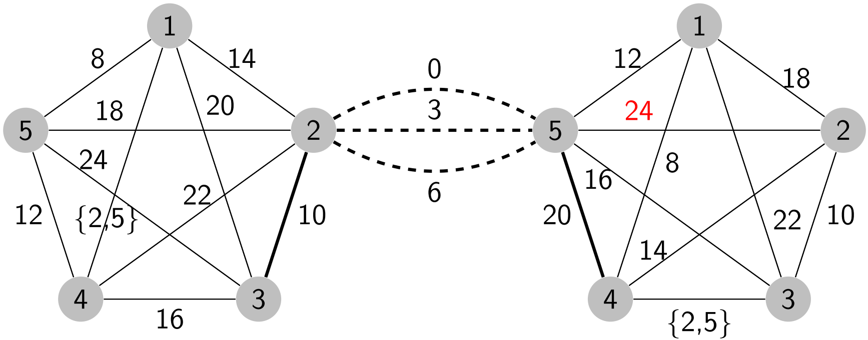
Note. In this case, $\lambda_e < \eta$ when $t = 4$ or $t \equiv 0 \pmod 3$, otherwise $\lambda_e = \eta$.

$t = 4$

$\lambda_e^{(2)} = 27$



$\lambda_e^{(2)} = 25$


1
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14
5
18
20
2
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12
{2,5}
10
4
3
16
0
3
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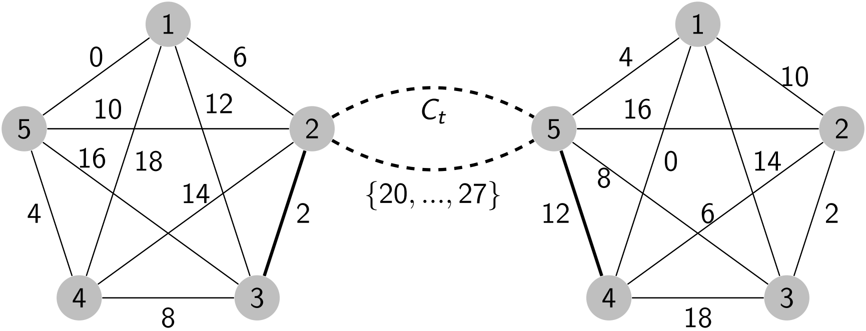
{2,5}

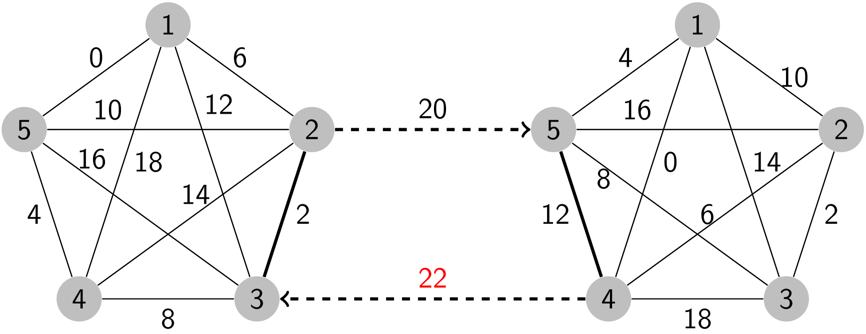
$t \not\equiv 0 \pmod 3$



$D_2(tK_5)$, t even

Theorem

$\lambda_e^{(2)}(D_2(tK_5)) = 23.$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14
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2
18

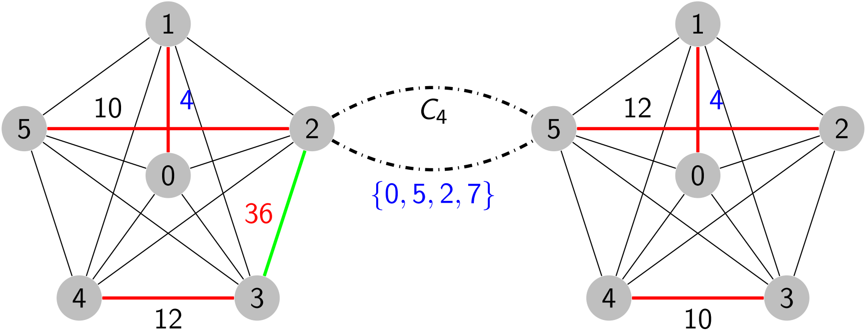
$D_1(tK_6)$, t even

Theorem

Let t be even.

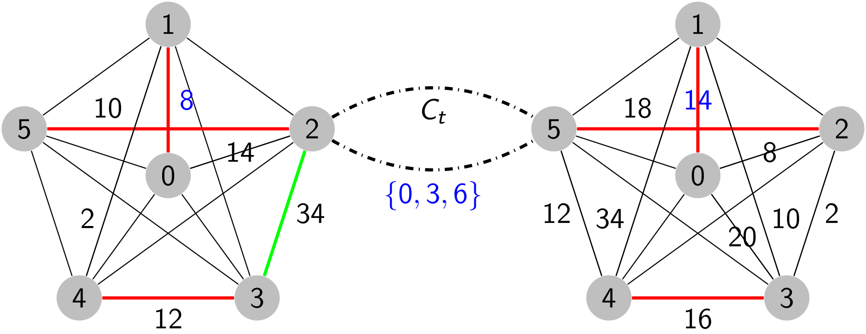
$$\lambda_e^{(2)}(D_1(tK_6)) = \begin{cases} 35 & t \equiv 0 \pmod 3, \\ 38 & t = 10, \\ 37 & \textit{otherwise}. \end{cases}$$

$t = 4$

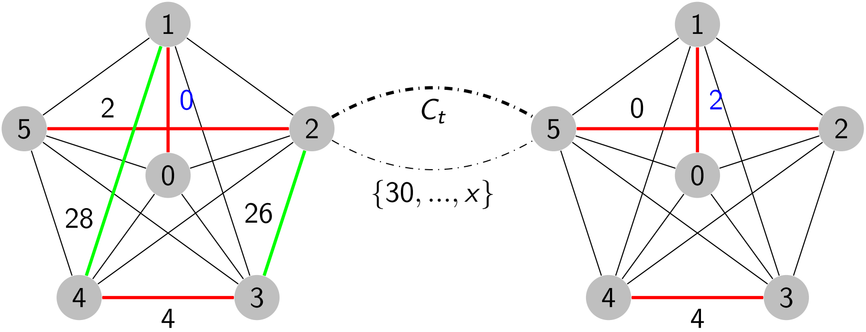


Note: $4 \rightarrow \{4, 9\}$

$t \equiv 0 \pmod 3$



$t \not\equiv 0 \pmod 3$

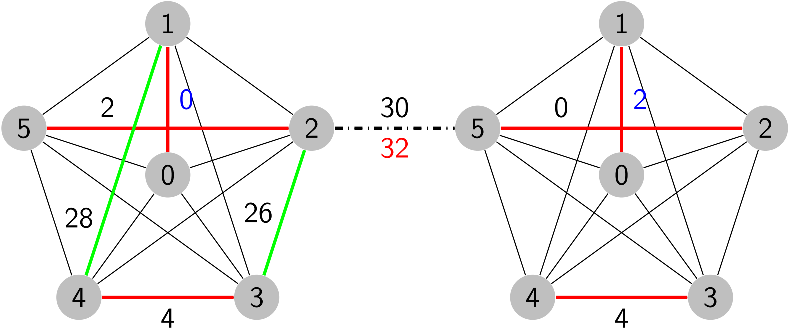


Note: $x = 38$ if $t = 10$, otherwise is 37

$D_2(tK_6)$, t even

Theorem

$\lambda_e^{(2)}(D_2(tK_6)) = 33.$



1
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28
26
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30
32
1
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2
0
4
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0
2
4

$\lambda_e^{(2)} = \eta$?

When $\lambda_e^{(2)} = \eta$?

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When $\lambda_e^{(2)} = \eta$?

- Suitably label each clique.

When $\lambda_e^{(2)} = \eta$?

- Suitably label each clique.
- There are only no-hole optimal labelings of the cycle.

Unsolved Cases

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1. t odd and $m = 5$

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1. t odd and $m = 5$
2. t odd , $t \equiv 0 \pmod 3$ and $m \geq 6$

The End

Thanks for your listening!