Partitioning complete graphs into monochromatic paths

Jun-Yi Guo
and
Zhen-Chun Chen

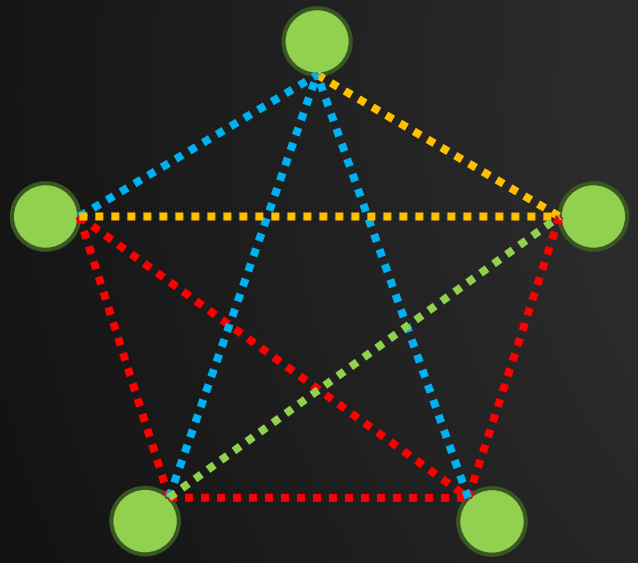
第十屆海峽兩岸
圖論與組合數學研討會

2019.08.22

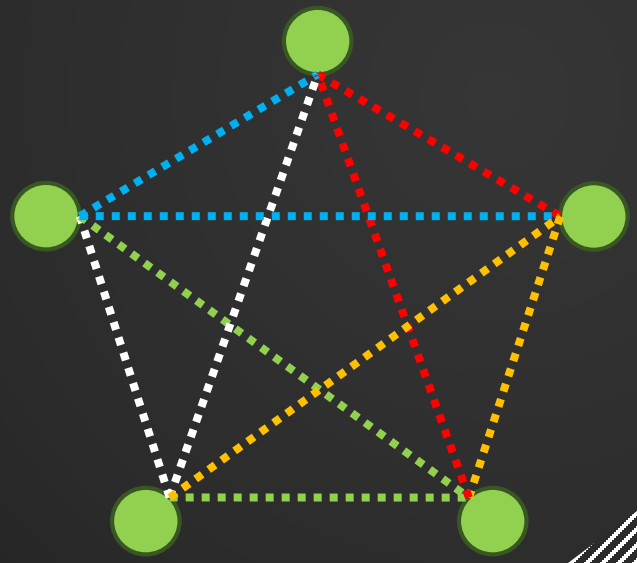
1 Basic Notations

Basic Notations

If the edges of the graph are colored with r colors is called **r-colored**.



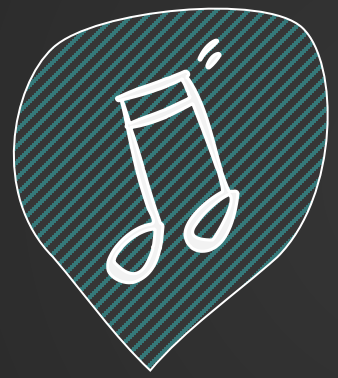
4-colored



5-colored

2 History

History



Monochromatic

Ramsey number

Heterochromatic

Rainbow

Theorem. For each **odd integer n ≥ 3**, Kn has **a proper n-edge-coloring** such that Kn can be partitioned into **(n-1)/2 isomorphic rainbow spanning cycles**.

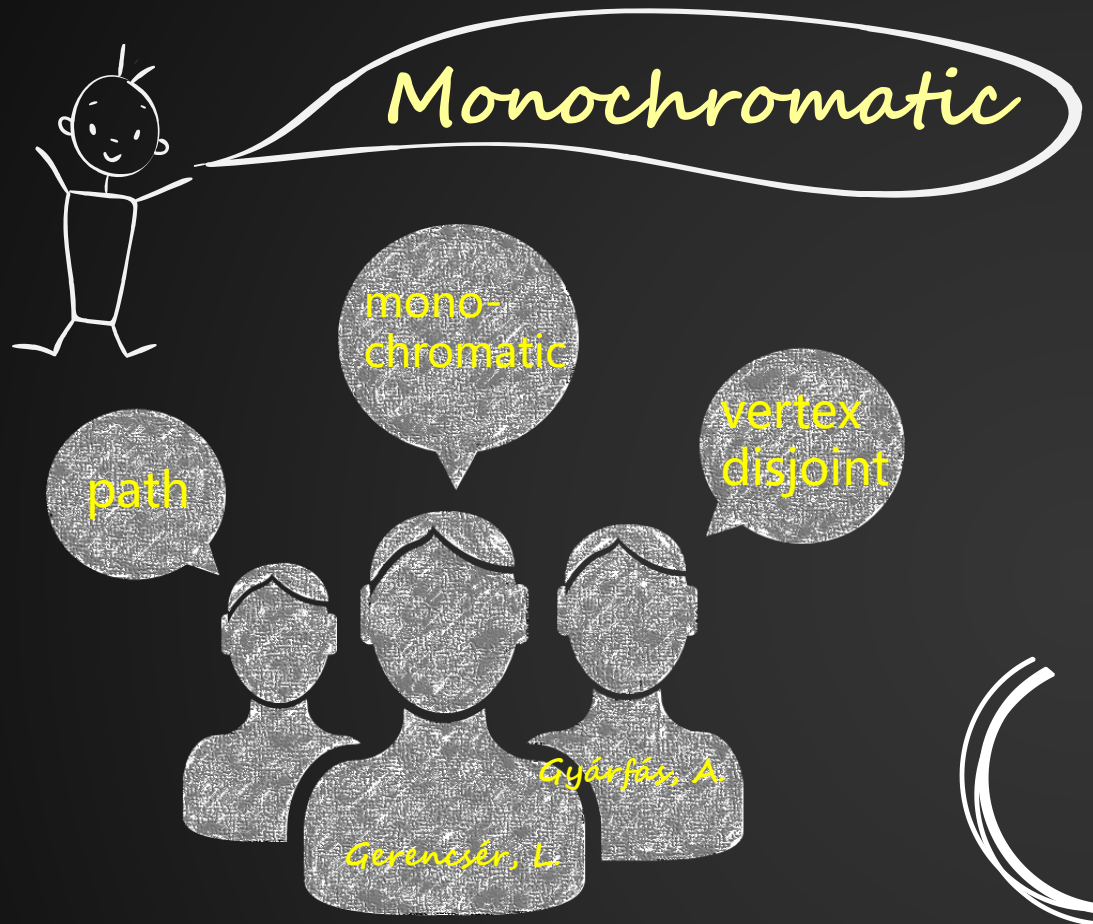
Hung-Lin Fu and Yuan-Hsun Lo, **Multicolored parallelisms of Hamiltonian cycles.** Discrete Math. 309 (2009), no. 14, 4871–4876.

Theorem. K_{2n} has **a proper (2n-1)-edge-coloring** such that K_{2n} can be partitioned into **n isomorphic rainbow spanning trees** except when n = 2.

S. Akbari, A. Alipour, Hung-Lin Fu, and Yuan-Hsun Lo, **Multicolored parallelisms of isomorphic spanning trees.** SIAM J. Discrete Math. 20 (2009), no. 3, 564–567.

tree
cycle
Hamiltonian path
Hamiltonian cycle
path
star

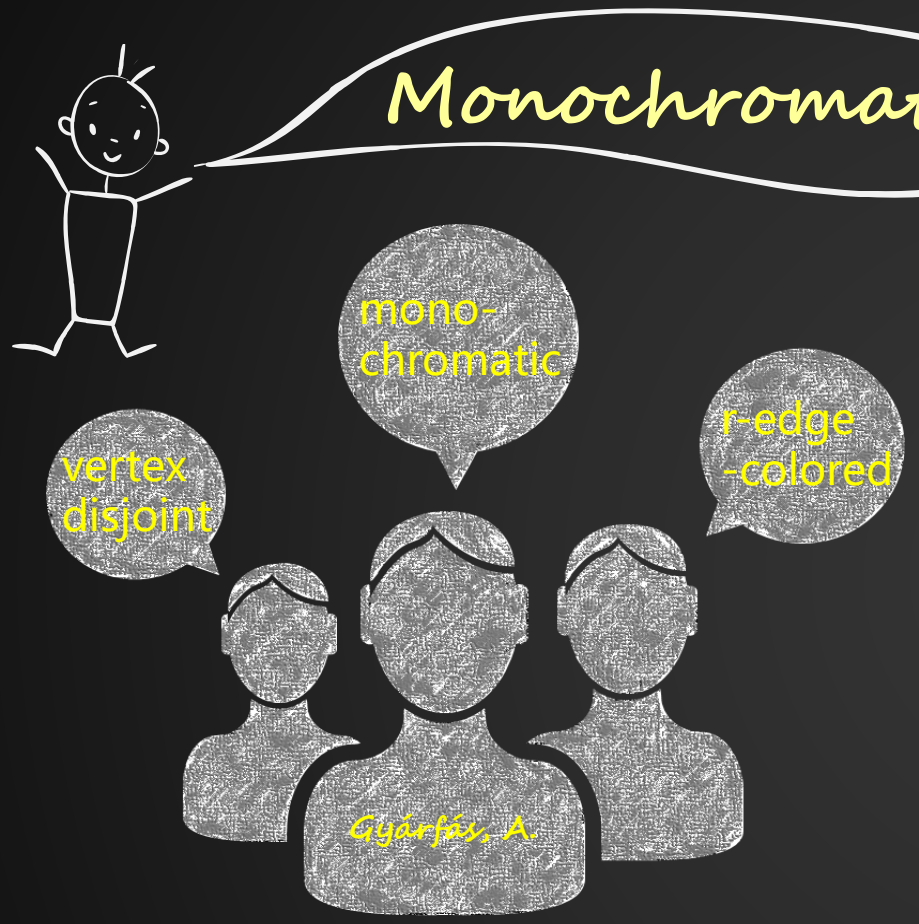
Complete graph



Theorem. Every **2-edge-colored complete graph Kn** contains either **a monochromatic Hamiltonian path** or **vertex-disjoint one red path and one blue path** that together cover the vertices of Kn.

Gerencsér, L., Gyárfás, A., On Ramsey-type problems. Ann. Univ. Sci. Bud. de Rol. Eötvös Sect. Math. 10 (1967), 167–170.

Gyárfás, A., Vertex coverings by monochromatic paths and cycles. J. Graph Theory 7 (1983), no. 1, 131–135.



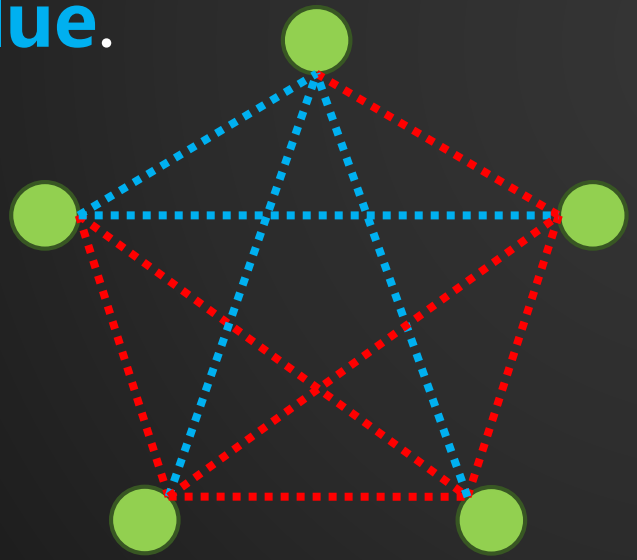
Conjecture. The vertices of every **r-edge-colored complete graph Kn** can be covered by **at most r vertex-disjoint monochromatic paths**.

Gyárfás, A., **Covering complete graphs by monochromatic paths.** in Irregularities of Partitions(1989), Algorithms and Combinatorics, Springer-Verlag 8, 89–91.

3 Known result

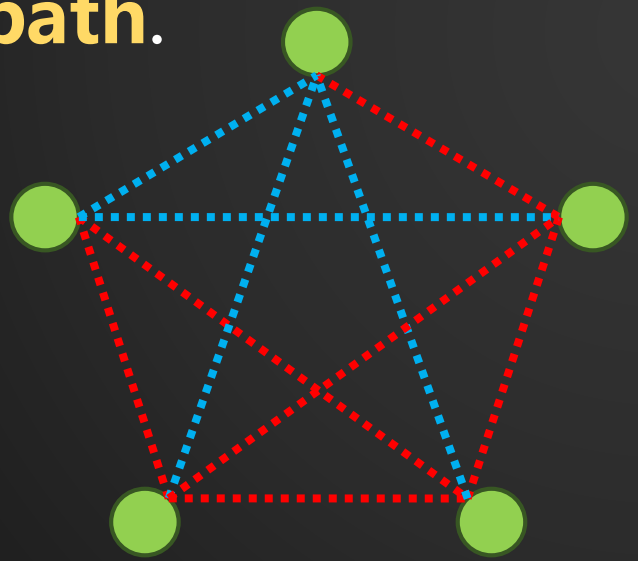
Basic Notations

A graph is **2-colored** if each edge is colored either **red** or **blue**.



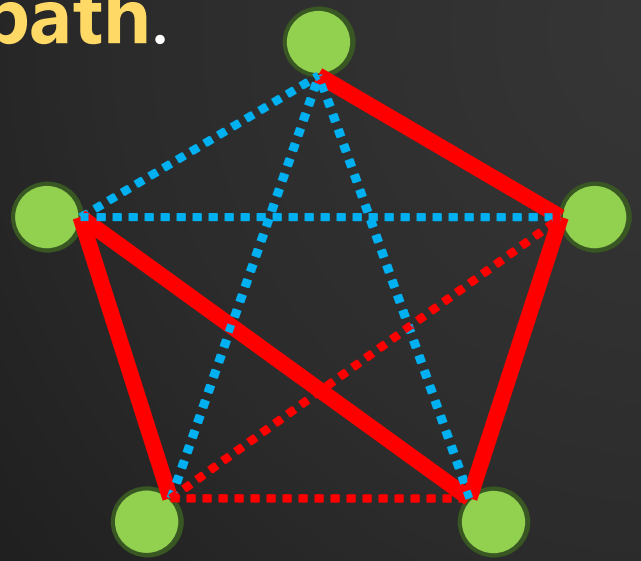
Basic Notations

A 2-colored path is called **simple** if **it is monochromatic** or **it is the union of a red and a blue path**.



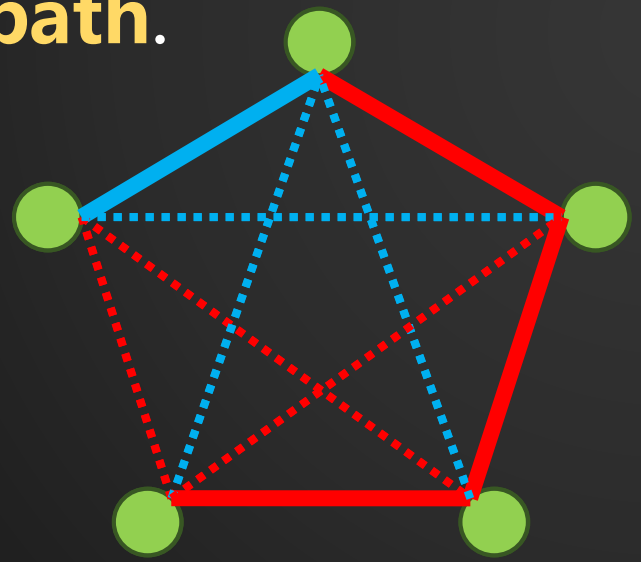
Basic Notations

A 2-colored path is called **simple** if **it is monochromatic** or **it is the union of a red and a blue path**.

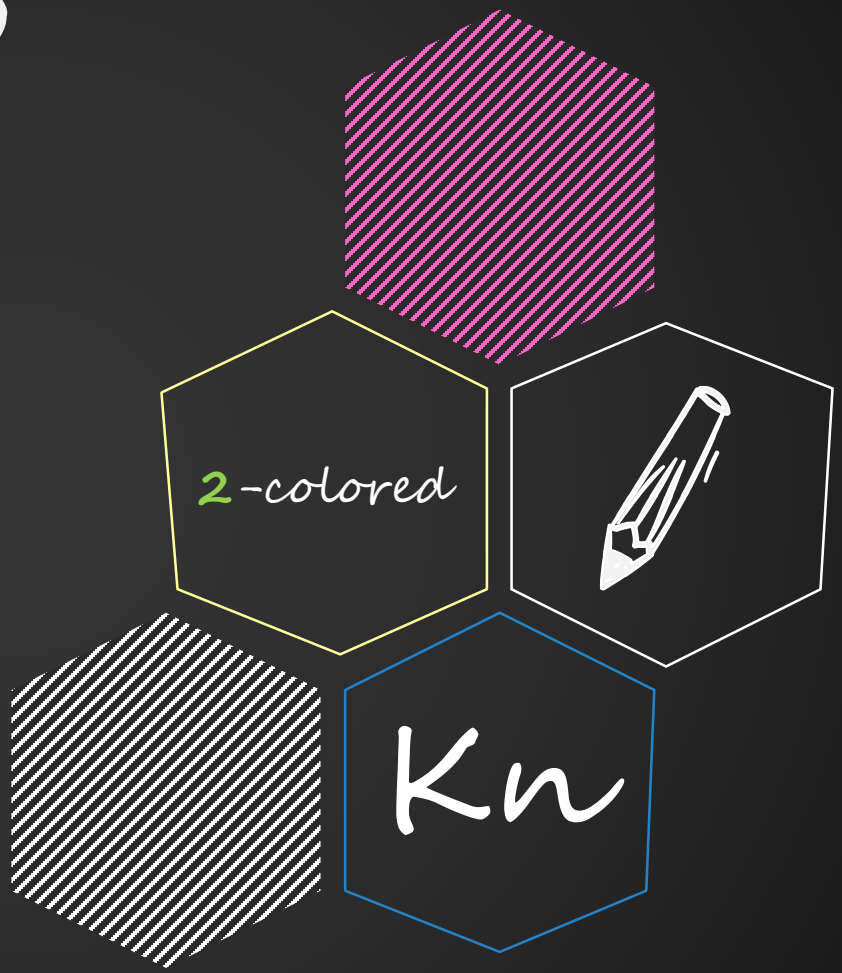


Basic Notations

A 2-colored path is called **simple** if **it is monochromatic** or **it is the union of a red and a blue path**.



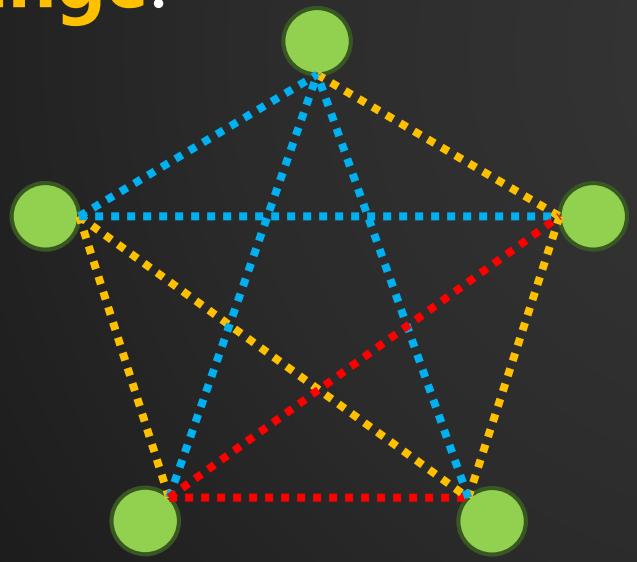
Theorem. A **2-colored complete graph Kn** contains **a simple Hamiltonian path**.

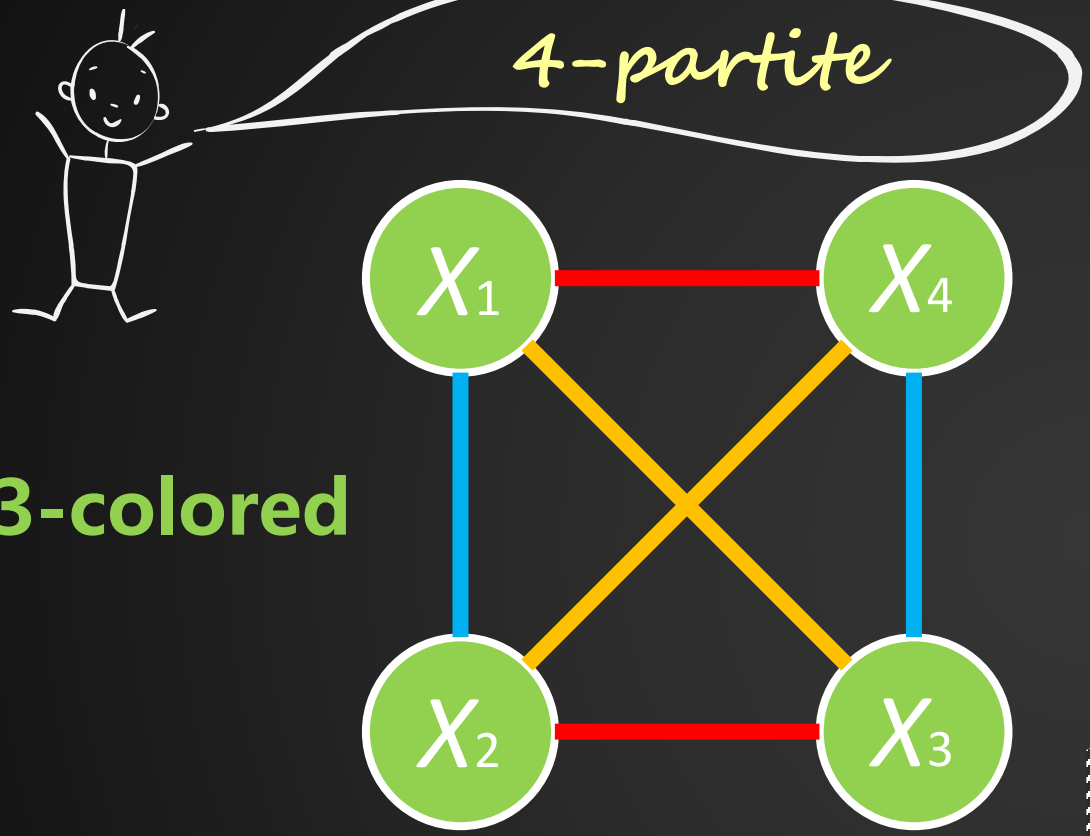


Gyárfás, A., Vertex coverings by monochromatic paths and cycles. J. Graph Theory 7 (1983), no. 1, 131–135.

Basic Notations

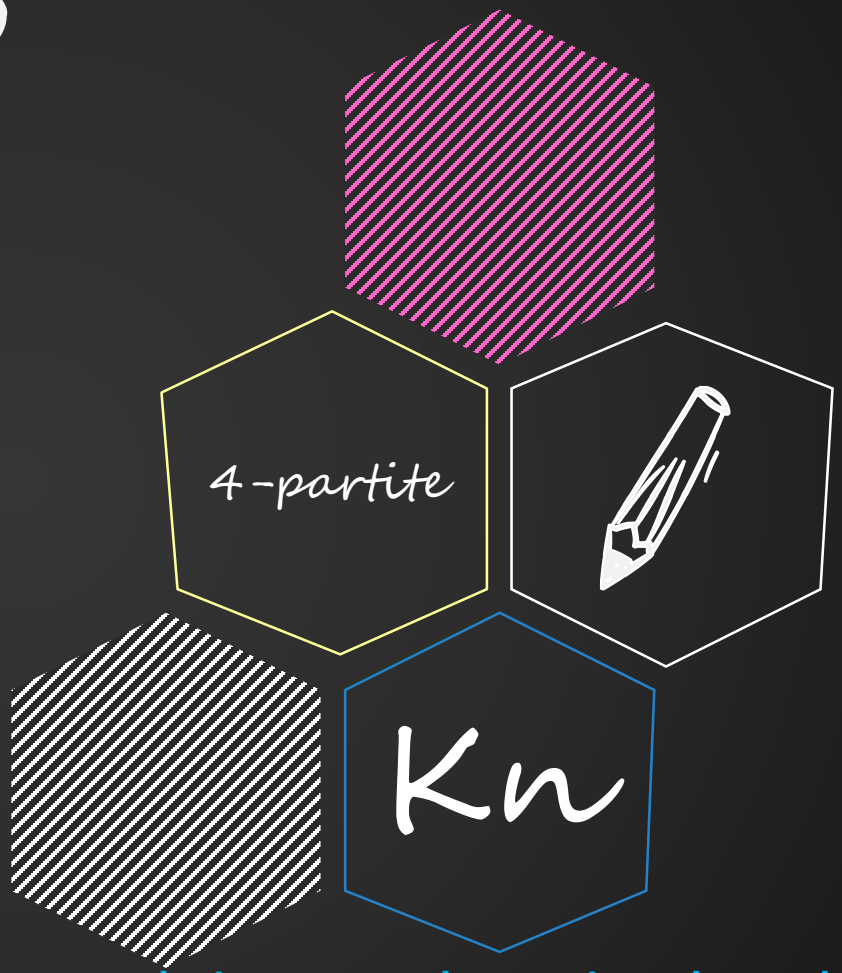
A graph is **3-colored** if each edge is colored **red**, **blue** or **orange**.



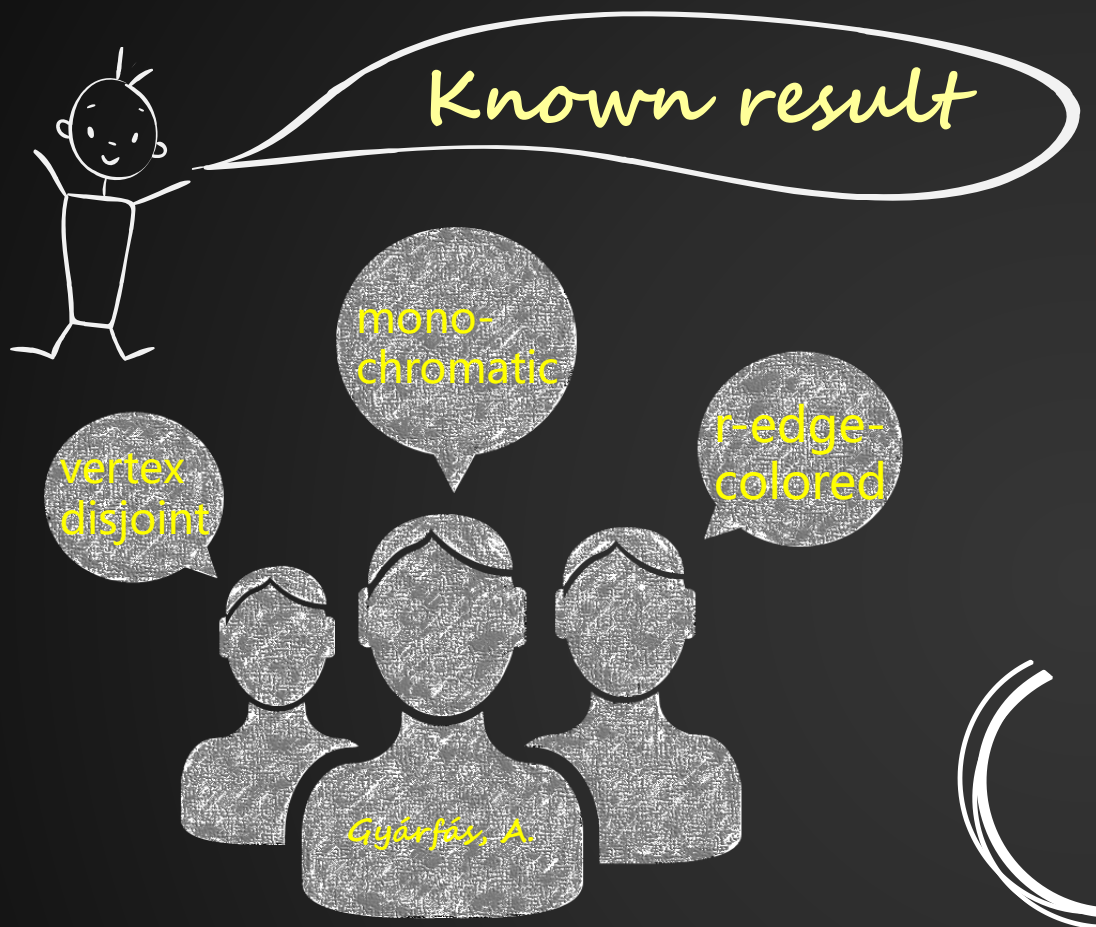


3-colored

The **edges** within the sets X_1, X_2, X_3, **and** X_4 can be coloured **arbitrarily**.



Alexey Pokrovskiy, Partitioning edge-coloured complete graphs into monochromatic cycles and paths.
J. Combin. Theory Ser. B 106 (2014), 70–97.



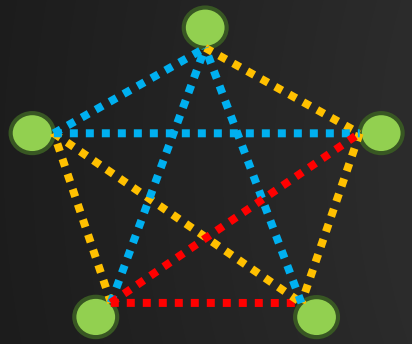
Theorem A. Suppose that the edges of Kn are coloured with **three colours** such that the colouring is **not 4-partite**. Then Kn can be **vertex-partitioned into three monochromatic paths** with **different colours**.

Alexey Pokrovskiy, **Partitioning edge-coloured complete graphs into monochromatic cycles and paths.** J. Combin. Theory Ser. B 106 (2014), 70–97.

4 Main result

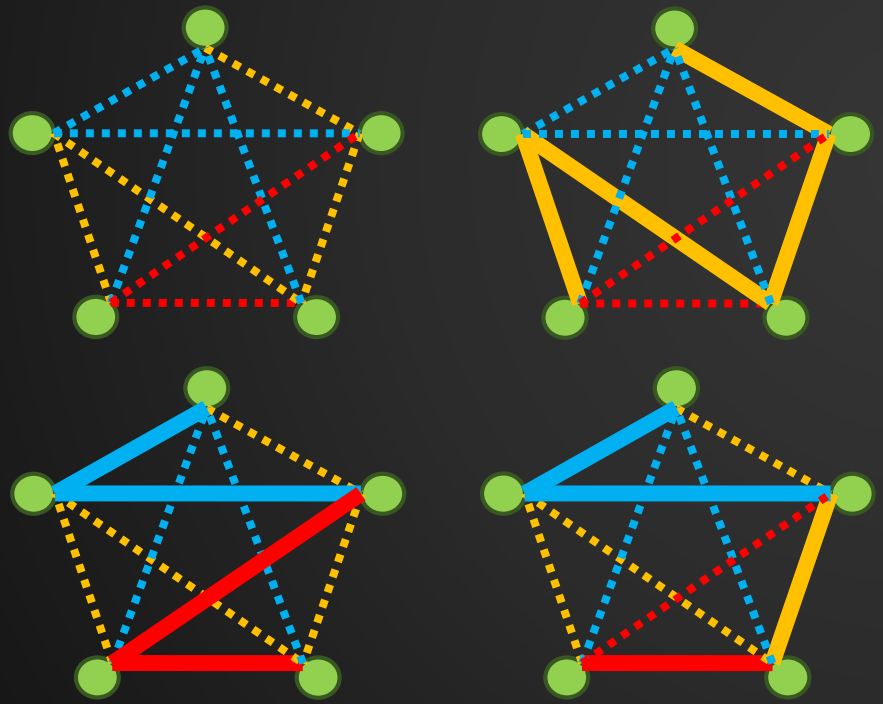
Basic Notations

A 3-colored path is **simple**.

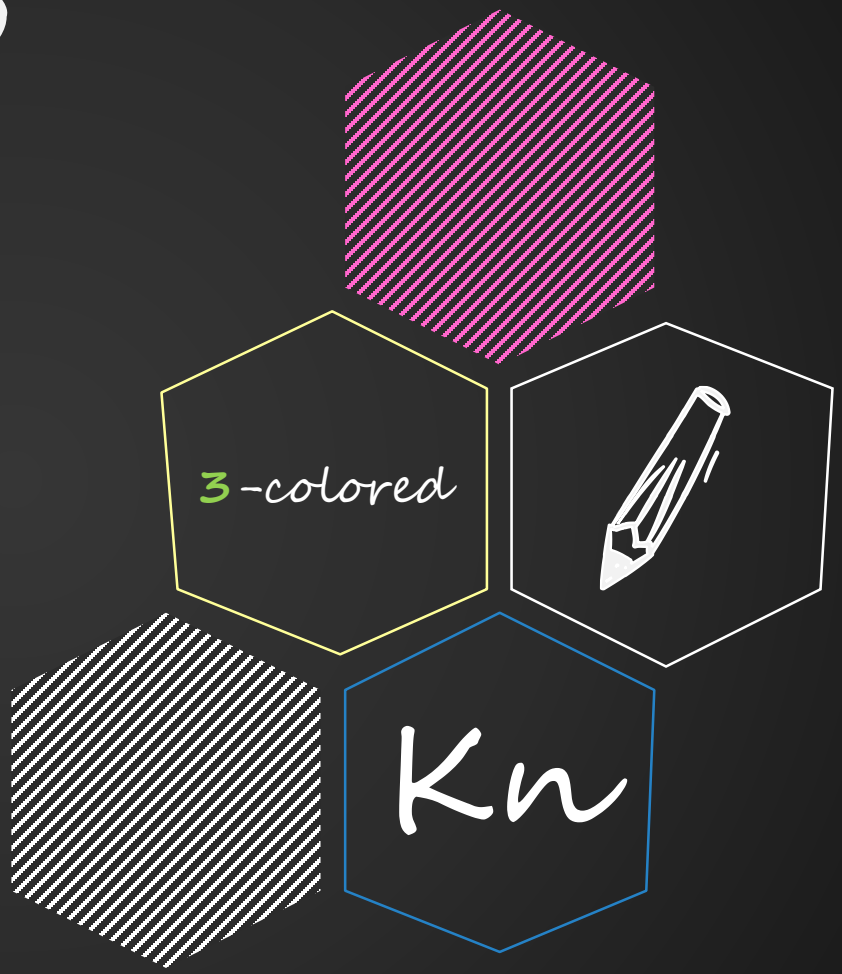


Basic Notations

A 3-colored path is **simple**.



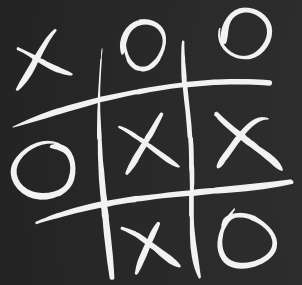
Theorem B. Suppose that the edges of Kn are coloured with **three colours** such that the colouring is **not 4-partite**. Then Kn contains **a simple Hamiltonian path**.



Main result

Theorem B. Suppose that the edges of Kn are coloured with **three colours** such that the colouring is **not 4-partite**. Then Kn contains **a simple Hamiltonian path**.

Theorem A. Suppose that the edges of Kn are coloured with **three colours** such that the colouring is **not 4-partite**. Then Kn can be **vertex-partitioned into three monochromatic paths** with **different colours**.



THANK YOU

Jun-Yi Guo
and
Zhen-Chun Chen

2019.08.22