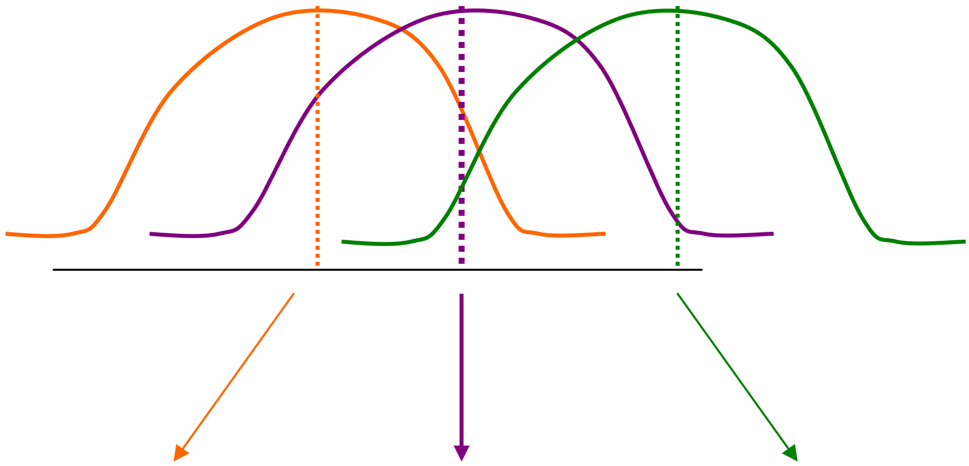
Lecture 08. Hypothesis Testing (I)

單樣本平均值之檢定; One-sample (t-) test

- 圖說



$N(\mu_1, \sigma^2)$ $N(\mu_2, \sigma^2)$ $N(\mu_3, \sigma^2)$

‘Hypothesis’ (or ‘Question’),

$\mu = \mu_1$ **?** $\mu = \mu_2$ **?** $\mu = \mu_3$ **?**

Data (sample)

× ××× × × × × × × ×

Data (sample)

× × × × × × × × × ×

- $Pr(\mathbf{X}|H_{\boldsymbol{\theta}})$?? $Pr(H_{\boldsymbol{\theta}}|\ \mathbf{X})$??

One-tailed or two-tailed tests??

General: Let $X_1, X_2, X_3, \ldots, X_n \sim N(\mu, \sigma^2)$

- **One-tailed test**

H_0: $\mu \leqq \mu_1$ (null hypothesis, 虛設假說) vs.

H_a: $\mu > \mu_1$ (alternative hypothesis, 對實假說)

- **Two-tailed t-test (T-檢定);**

H0: $\mu = \mu_1$ vs. **Ha: $\mu \neq \mu_1$**

- **To test the hypothesis based on the data (evidence) collected !!**

Procedure

- **To construct the test statistic:**
  (difference of sample mean and the hypothesized mean) ÷ std. err.
- **Sampling distribution of the test statistic under H_0 :**
  $t_{(n-1)}$, degree of freedom, df,=n-1 ，(variance unknown)
- **Significance level (1-α)**
- **Type I error (α) and p-value**

Data, hypothesis and test statistic

$X_1, X_2, X_3, \ldots, X_n \sim N(\mu, \sigma^2)$, σ^2 **is an unknown parameter which needs to be estimated. In order to test the following two-sided (two-tailed) hypothesis**

H_0: $\mu = \mu_0$ vs. **Ha: $\mu \neq \mu_0$**,

We use the sample mean $\overline{X}$ to see whether or not its realization looks significantly different from μ_0.

Deduction

$$X_1, X_2, X_3, \ldots, X_n \sim N(\mu, \sigma^2)$$

$\Longrightarrow$ $\bar{X} \sim N(\mu, \sigma^2/n)$,

sampling distribution of the sample mean

$\Longrightarrow$ $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim$ **N(0, 1), standardization**

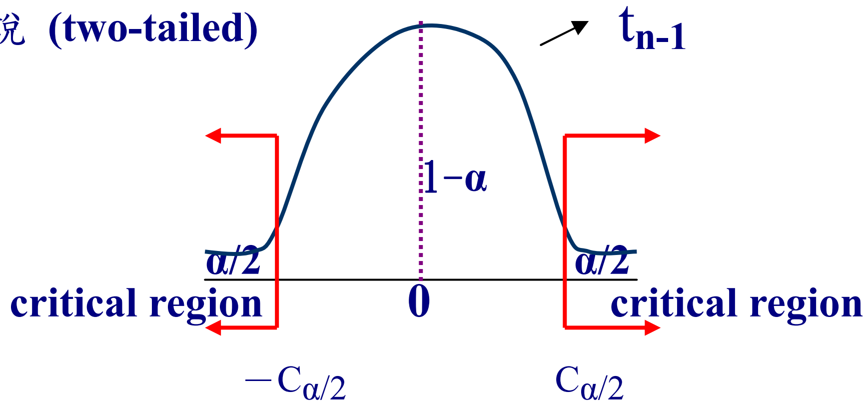
$\Longrightarrow$ **under** H_0, $\dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \sim$ **N(0, 1), if** σ^2 **is known**

$\Longrightarrow$ $\dfrac{\bar{X}-\mu_0}{\hat{\sigma}/\sqrt{n}} \sim t_{(n-1)}$, $\hat{\sigma} = s$ $(\hat{\sigma}^2 = s^2)$

$s^2 = \dfrac{\Sigma(X_i - \bar{X})^2}{n-1}$, **if** σ^2 **is unknown**

If the value (realization) of the statistic $\dfrac{\bar{X}-\mu_0}{\hat{\sigma}/\sqrt{n}}$ **is extremely large (positive or negative), we tend to judge that** H_0 **is wrong and** H_a **is correct. In statistical terminology, we say we reject or do not reject** H_0 **whenever the absolute value of** $\dfrac{\bar{X}-\mu_0}{\hat{\sigma}/\sqrt{n}}$ **is large or small, under the** ***α*****-level of significance.** **('two-sided' problem)**

圖說 (two-tailed)



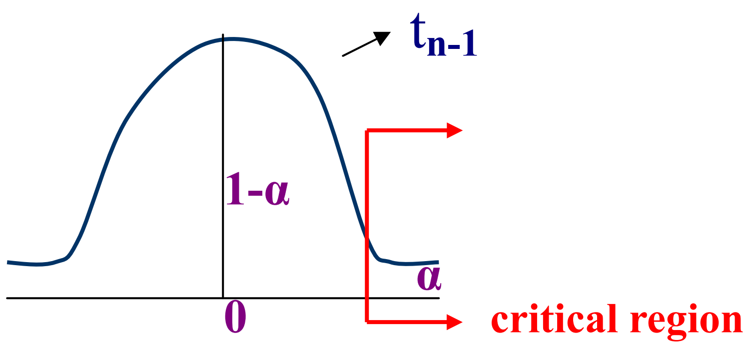
That is, we will reject H_0 if $\left| \dfrac{\bar{X}-\mu_0}{\hat{\sigma}/\sqrt{n}} \right| > C_{\alpha/2}$

(Note: $P\left(\left| \dfrac{\bar{X}-\mu_0}{\hat{\sigma}/\sqrt{n}} \right| > C_{\alpha/2}\right)=\alpha$, $P\left(\left| \dfrac{\bar{X}-\mu_0}{\hat{\sigma}/\sqrt{n}} \right| < C_{\alpha/2}\right)=1-\alpha$)

Equivalently, we will reject H_0 if

$$\bar{X} > \mu_0 + C_{\alpha/2}\frac{\hat{\sigma}}{\sqrt{n}} \quad \text{or} \quad \bar{X} < \mu_0 - C_{\alpha/2}\frac{\hat{\sigma}}{\sqrt{n}}$$

One-tailed :



Example 1. (高血壓抽煙男性膽固醇平均值與一般族群之比較)

Hypothesis $H_0: \mu = 211$ vs. $H_a: \mu \neq 211$, (Two-tailed)

Data: (Note that only sample mean and sample standard deviation is used in a t-statistic.)

Random sample: 12 hypertensive smokers

Mean serum cholesterol level $\bar{X}$=217 mg/100ml;

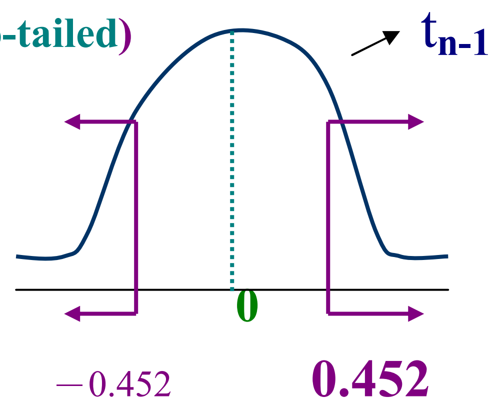
Sample standard deviation s=46 mg/100ml;

Significance level α=0.05;

$$\frac{\bar{X}-\mu_0}{\hat{\sigma}/\sqrt{n}} = \frac{217-211}{46/\sqrt{12}} = \mathbf{0.452} < 2.201 = t_{11,0.025}$$

$$-2.201 < \mathbf{0.452} \Longrightarrow \text{do not reject } H_0$$

P-value (two-tailed)



p=Pr(t_{n-1}<-0.452 or >0.452)>0.20, approximately, from the t-table.

(WHY??) (Exactly, p=0.66)

Note: P-value 之計算須視 H_0 與 H_a 之相對'方向'來決定

結論: 此 12 名高血壓抽煙男性之膽固醇平均值與一般族群平均值比較，統計上差異不顯著。